THE BULLETIN OF THE

USER GROUP

+ CAS-TI

Contents:

Francisco Marcelo Fernadez wrote:

Dear Josef,

I found a problem when using Derive second derivative. Is any sound reason for this behaviour?. I am attaching a Derive file.

Best regards,

Marcelo

$$g(x) :=$$

$$\frac{d}{dx}\frac{d}{dx}(\text{EXP}(-\alpha \cdot x^2)\cdot g(x)) = e^{-\alpha \cdot x^2}\cdot(g''(x) - 2\cdot\alpha\cdot(x\cdot g'(x) + g(x))) + e^{-\alpha \cdot x^2}\cdot(4\cdot\alpha^2\cdot x^2\cdot g(x) - 2\cdot\alpha\cdot x\cdot g'(x))$$

but:

$$\left(\frac{d}{dx}\right)^2(\text{EXP}(-\alpha \cdot x^2)\cdot g(x)) = \left(\frac{d}{dx}\right)^2\left(e^{-\alpha \cdot x^2}\cdot g(x)\right)$$

Dear Josef,

The problem that I sent you before only affects Derive 6. It does not appear in Derive 5 as shown in the attached file.

Best regards,

Marcelo

$$g(x) :=$$

$$\left(\frac{d}{dx}\right)^2\left(\hat{e}^{-\alpha \cdot x^2}\cdot g(x)\right) = \hat{e}^{-\alpha \cdot x^2}\cdot(g''(x) - 2\cdot\alpha\cdot(2\cdot x\cdot g'(x) + (1 - 2\cdot\alpha\cdot x^2)\cdot g(x)))$$

Michel Beaudin's comment:

This problem is really a strange behavior of Derive. In fact, if you replace g(x) by any known function (cos(x), ln(x), x^4, ...), the notation (d/dx)^2 works (but I find this notation so ugly !). I don't know where is the problem. Was it a bug from Derive no one has seen during all these years ? If I find something in the upcoming days, I let you know. Take care,

Dear Josef, I don't have Derive 5 version but if you have it, please check the following: consider the 2X2 matrix A:= [1,2;3,4] and simplify exp(A). In Derive 6, this yields a wrong answer (the exponential of a square matrix IS NOT the matrix of the exponential of each entry unless the matrix is already diagonal). This a bug in Derive 6 and I think it was not a bug in Derive 5 (Derive 5 was not simplifying).

Michel

I „asked" wxMaxima and Mathematica and found the reason for the "bug". There is an important difference between exp(matrix) and matrixexp(matrix). The latter is not implemented in DERIVE, but – as Michel wrote later – you do not need to do without it. One can do the matrix exponential with DERIVE and TI-NspireCAS as well. It will be described in the next DNL See the difference next page, Josef

wxMaxima with exp(M) and matrixexp(M):

```
(%i2)    a:matrix([1,2],[3,4]);
```

$$(a) \quad \begin{bmatrix} 1 & 2 \\ 3 & 4 \end{bmatrix}$$

```
(%i3)    exp(a);
```

$$(\%o3) \quad \begin{bmatrix} \%e & \%e^2 \\ \%e^3 & \%e^4 \end{bmatrix}$$

```
(%i4)    matrixexp(a);
```

$$(\%o4) \quad \begin{bmatrix} -\dfrac{\%e^{5/2-\frac{\sqrt{33}}{2}}\left(\left(\sqrt{33}-11\right)\%e^{\sqrt{33}}-\sqrt{33}-11\right)}{22} & \dfrac{\%e^{5/2-\frac{\sqrt{33}}{2}}\left(2\sqrt{33}\,\%e^{\sqrt{33}}-2\sqrt{33}\right)}{33} \\[4ex] \dfrac{\%e^{5/2-\frac{\sqrt{33}}{2}}\left(\sqrt{33}\,\%e^{\sqrt{33}}-\sqrt{33}\right)}{11} & \dfrac{\%e^{5/2-\frac{\sqrt{33}}{2}}\left(\left(\sqrt{33}+11\right)\%e^{\sqrt{33}}-\sqrt{33}+11\right)}{22} \end{bmatrix}$$

Dear DUG Members,

Due to the fact that this DNL is pretty full, I don't take much space for my letter. First of all, it is a great concern for me to commemorate Albert Rich, one of the "Fathers of DERIVE".

As you will see, this is a very colorful issue, because we have two articles treating fractals – one of them using DERIVE and the other one working with TI-NspireCAS based on a LUA program. Roman Hašek had in his book[1] a short chapter on Julia Sets and Newton iteration with two figures in gray scale. When I tried his functions, I got very nice colored plots – see on page 8.

A research in the web for fractal generators – caused by a fascinating article on "Mandelpinskis" in the "Spektrum der Wissenschaft 7/23" - led me to fractal zoomer.tns programmed by Jens Kolbinger. The figures presented on the download site were so tempting, that it was a must for me to download the program and make my own experiments. It was not so easy to reproduce the Mandelbrot Sets. All programs, which I tried did not offer the opportunity to edit one's own iteration function. I inspected the LUA code – and was lucky enough to change the code in a way that I could (and you can, too) plot not only the "Little Apple Man", but also other Mandelbrot Sets (page 35).

From earlier DNLs you might know that Wolfgang Alvermann is very often inspired by mathematical and geometrical puzzles. He infects me with his passion (page 23).

Finally, I'd like to draw your attention to GXWeb, which is explained by Steve Arnold on websites, informative and full of content (page 21).

Best regards and have a good time

Josef

[1] Hašek, R. *Užití Derive ve výuce matematiky*. České Budějovice: Europeon, a. s., 2007.

The *DERIVE-NEWSLETTER* is the Bulletin of the *DERIVE* & CAS-*TI User Group*. It is published at least four times a year with a content of 40 pages minimum. The goals of the *DNL* are to enable the exchange of experiences made with *DERIVE, TI-CAS* and other CAS as well to create a group to discuss the possibilities of new methodical and didactical manners in teaching mathematics.

Contributions:
Please send all contributions to the Editor. Non-English speakers are encouraged to write their contributions in English to reinforce the international touch of the *DNL*. It must be said, though, that non-English articles will be warmly welcomed nonetheless. Your contributions will be edited but not assessed. By submitting articles, the author gives his consent for reprinting it in the *DNL*. The more contributions you will send, the more lively and richer in contents the *DERIVE* & CAS-*TI Newsletter* will be.

Next issue: December 2023

Preview: Contributions waiting to be published

Some simulations of Random Experiments, J. Böhm, AUT, Lorenz Kopp, GER
Wonderful World of Pedal Curves, J. Böhm, AUT
Simulating a Graphing Calculator in *DERIVE*, J. Böhm, AUT
Logos of Companies as an Inspiration for Math Teaching
Exciting Surfaces in the FAZ, BooleanPlots.mth, P. Schofield, UK
Old traditional examples for a CAS – What´s new? J. Böhm, AUT
Tutorials for the NSpireCAS, G. Herweyers, BEL
Dirac Algebra, Clifford Algebra, Vector-Matrix-Extension, D. R. Lunsford, USA
Another Approach to Taylor Series, D. Oertel, GER
Charge in a Magnetic Field, H. Ludwig, GER
More Applications of TI-Innovator[TM] Hub and TI-Innovator[TM] Rover
Surfaces and their Duals, Cayley Symmetroid, J. Böhm, AUT
A Collection of Special Problems, Puzzles, W. Alvermann, GER
DERIVE Bugs? D. Welz, GER
Tweening & Morphing with TI-NspireCX-II-T, J. Böhm. AUT
The Gap between Poor and Rich, J. Böhm, AUT
More functions from M. Myers and from Bhuvanesh's Mathtools-library
TaxiCab Conics, Two alternate Approaches to Conics, R. Haas, USA
QR-Code light, Random numbers following a given distribution
Quartiles, Numeros Primos, F. de Jesús Martínez Vargas, Mexico
Orthogonal trajectories to isoptics of ovals, M. Skrzypiec, PL
153 is another Special Number
The Apollonian Cubic
Contributions from E. Varbanova, Th. Dana-Picard and others

Albert D. Rich III (April 26, 1949 - August 11, 2023)

Rich, TIME 2004 Conference, Montréal, Québec, Canada, July 15-18, 2004; photo by Karsten Schmidt.
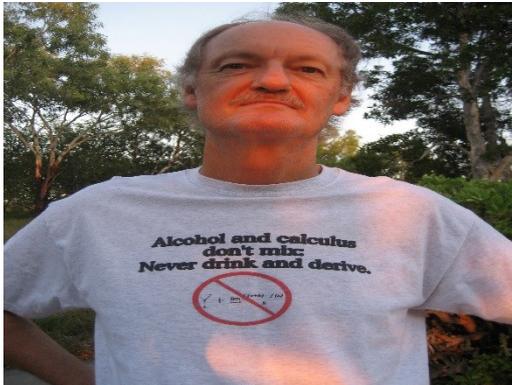
Don't Drink and DERIVE; 2009.

Albert and Joan Rich at a New York party; 2018.

Albert Rich (age 74), co-developer of the popular Derive computer algebra system, and before that of muLisp and muMath, died on Friday 11 August, after a short illness. After Derive was purchased by Texas Instruments, Albert devoted himself to Rubi, a public domain system for indefinite integration. Albert did not enjoy attending conferences, and preferred to let his website rulebasedintegration.org speak for him. He will be missed by family, friends and colleagues.

--- David Jeffrey

Albert D. Rich autobiographical note (October 18, 1998)

[Quoted from: Michael J. Wester (editor), *Computer Algebra Systems: A Practical Guide*, John Wiley & Sons, Chichester, United Kingdom, ISBN 0-471-98353-5, pp. 412--413, 1999.]

Albert D. Rich III was born in Altadena, California on April 26, 1949 to Albert D. Rich II and Alberta W. Rich. Being almost ten years younger than his three siblings, he was probably an unexpected surprise to his parents. At age 7, the family moved to Nacogdoches, Texas where Albert was raised. His father was a chemical engineer in the vegetable oil bleaching clay industry. Because of his mother's love of travel, she started a successful travel agency. Albert was active in the Boy Scouts, but hated competitive sports.

Albert graduated from Nacogdoches High School in 1967, and won a Navy ROTC scholarship to the University of Texas. There he learned various programming languages, but immediately latched onto LISP as the language most in tune with his way of thinking.

After studying Bertrand Russell's *Principia Mathematica*, Albert became interested in automating the process of proving theorems and generating interesting new ones. He collaborated with Laurant Siklossy, a professor of computer science, on a paper about a unification algorithm invented by Albert for proving theorems in the propositional calculus. In 1971, he received a BA in mathematics from UT.

Upon graduation, Albert was commissioned as an officer in the US Navy. After a year of Navy nuclear power school, he served the remainder of his military commitment on the U.S.S. James Monroe, a ballistic-missile submarine home-ported in Honolulu, Hawaii. In June of 1976, he was honorably discharged in Newport-News, Virginia, but immediately went racing back to Hawaii.

Having some ideas for theorem proving that he wanted to implement, Albert bought a kit for an IMSAI 8080 computer and soldered it together. This S-100 computer had 4K memory boards costing about $500 each (several 1000 times more per byte than now). Having only enough money to populate one board with just 2K bytes of memory, Albert gained a real appreciation for a byte: a concept seeming unknown to the current generation of programmers.

The next step was to implement a LISP interpreter for the IMSAI. By hand-assembling machine-code and entering it from the front panel switches, ``first light'' for the interpreter was achieved by the end of 1976 (about the same time Bill Gates and Paul Allen were writing their Basic interpreter for the Altair computer).

In 1977, David Stoutemyer, a professor at the University of Hawaii, heard that Albert had a LISP system running on an Intel 8080 based computer. Having done pioneering work in the field of computer algebra, David was interested in bringing these capabilities out of the M.I.T. and Stanford closets and making them available to the masses. Since (as Albert is still convinced) LISP is the best language for implementing computer algebra systems, they began work on the first and only CAS that would run on CP/M based machines. In 1979, Albert and David formed Soft Warehouse, Inc. and began selling muMATH, the predecessor to DERIVE. And the rest is history!

Michel Beaudin wrote:

Dear friends, the passing of Albert has affected everyone. On my side, the best way to honor his memory is to talk and write about integrals. So, here is what I have been doing for the past 5 weeks.

Réflexion sur les tables de dérivées et d'integrales.

We will publish this at least partially in the next newsletters. Josef

We have two contributions treating fractals in this DNL. I came across two very nice fractalizers. I assume that you know the "dinosaur" `Fractint`. Here are the links to other ones:

http://www.fractalizer.de

https://xenodream.com/jux.htm
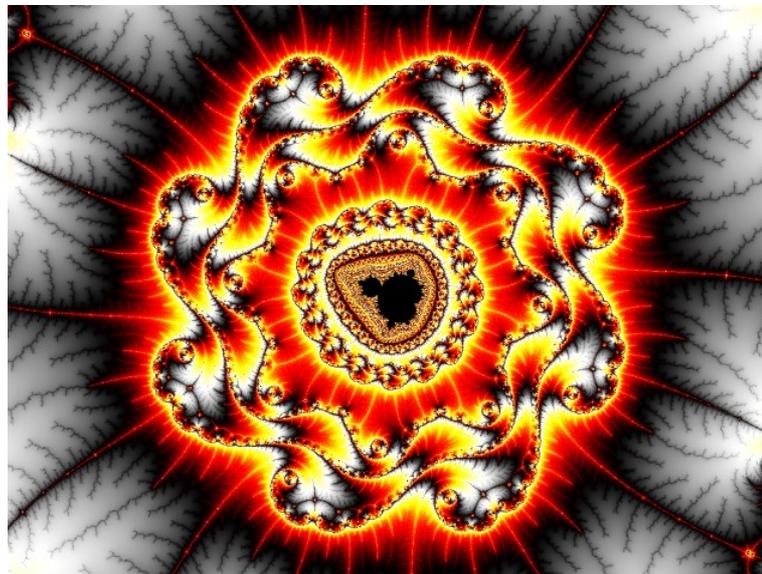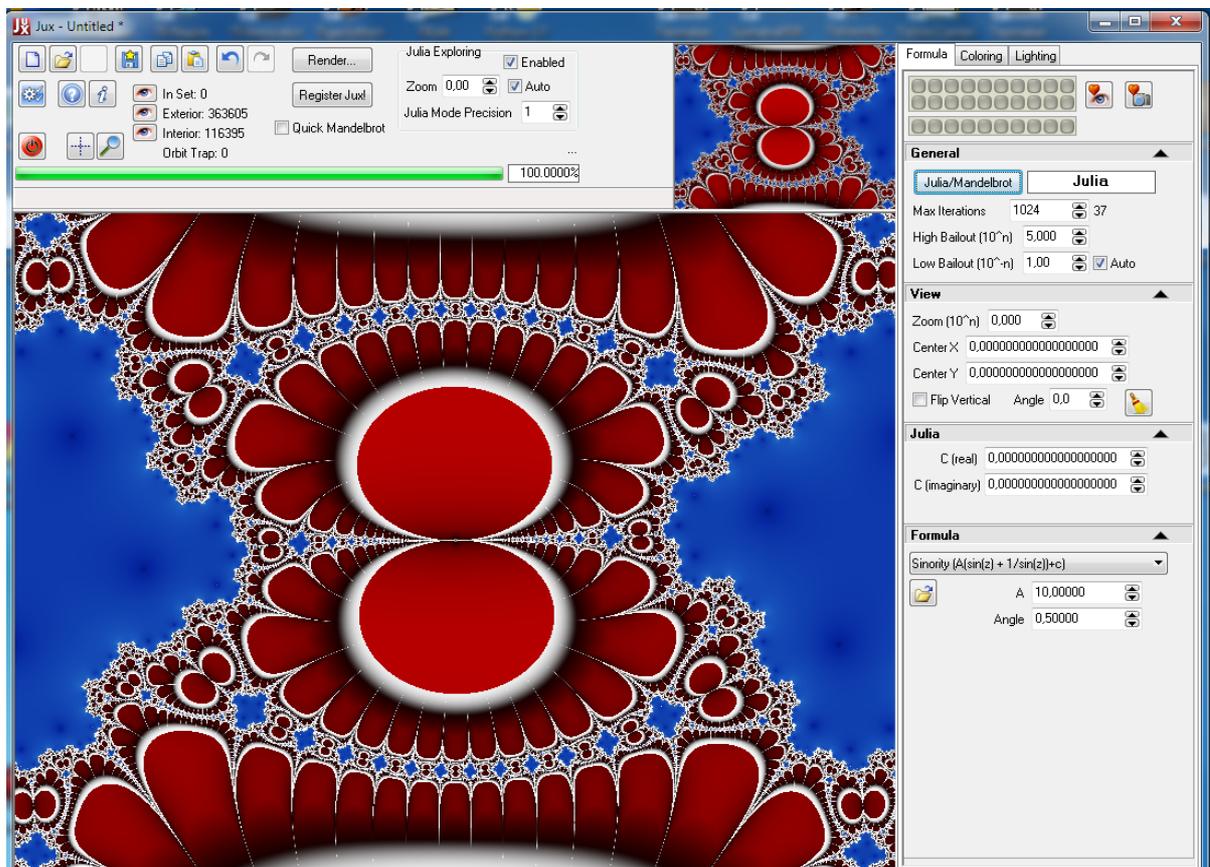
`Jux 4.0` offers a long list of types of iteration formulas see here $f(z) = A (\sin(z)+1/\sin(z))+c$.

Julia meets Newton

Roman Hašek, Czech Republic,Co-Author: Josef Böhm, Austria

Several years ago, I was invited to give a talk at the University in Ceske Budejovice, Czech Republic. On this occasion Roman gave me his book *Užití DERIVE ve výuce Matematiky* (*The use of DERIVE in mathematics teaching*)[1]. Although the book is only available in the Czech language, it is worth giving consideration to a number of interesting examples included in it how to use DE-RIVE.In one of the last chapters Roman deals with the DERIVE specialties: ITERATE, ITERATES and the VECTOR command.

I wrote to Roman and asked him for permission to show this part of his book together with some more graphs. This is what he answered:

Dear Josef,
Those are really pretty pictures you created. It reminds me again how smart DERIVE is. It is definitely worth showing in the Newsletter. Please, do it!
All the best,
Roman

After confirming his interest in publishing an article relating to this chapter of the book, Roman sent me an English translation of the text, which forms the basis of this article, in which I further develop some of his solutions below. Roman insisted to add me as 2nd author but in fact I just played with his functions.

So, here it is:

Chapter 26.3 Fractal phenomena in the complex number plane

Example 26.1: Julia Sets

Display the Julia set for $c = 0.2 + 0.4i$ in a circular region of the complex plane centered at the origin and with radius 2.

```
[PrecisionDigits := 10, NotationDigits := 10]
```

$$julia(f, c) := VECTOR\left(|ITERATE(f + c, z, x + i \cdot y, 4) - x - i \cdot y| > m \land |x + i \cdot y| < 2, m, 0, 3, \frac{1}{8}\right)$$

$$julia(z^2, 0.2 + 0.4 \cdot i)$$
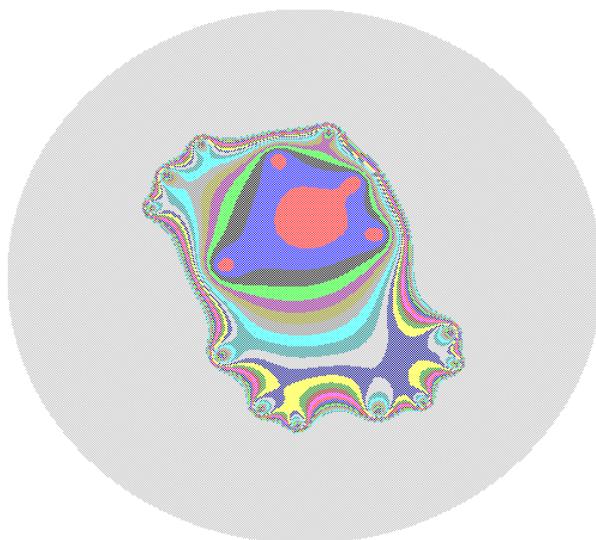
Roman explains the VECTOR-command as follows:

> The result is a vector whose components are the relations of variables x and y, each de-fining those points of the given region of the complex plane whose image distance from the initial position is greater than the corresponding value of the parameter m after four iterations of the function $f(z) = z^2 + c$.

```
       2
julia(z , −0.1234 + 0.74·i)
```
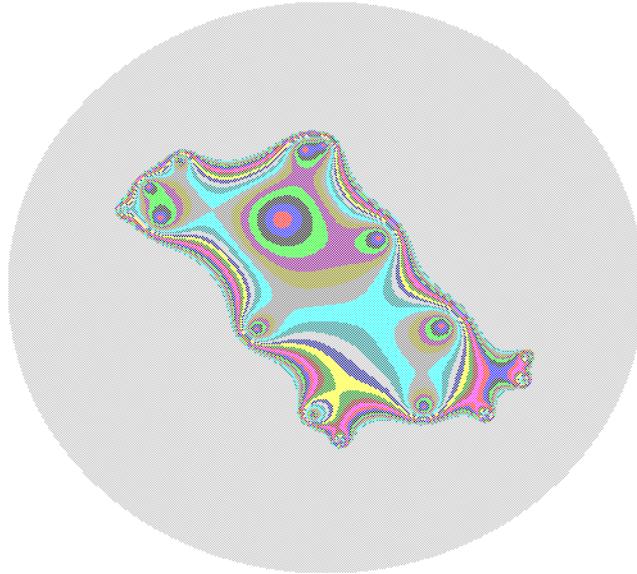

We can zoom in:
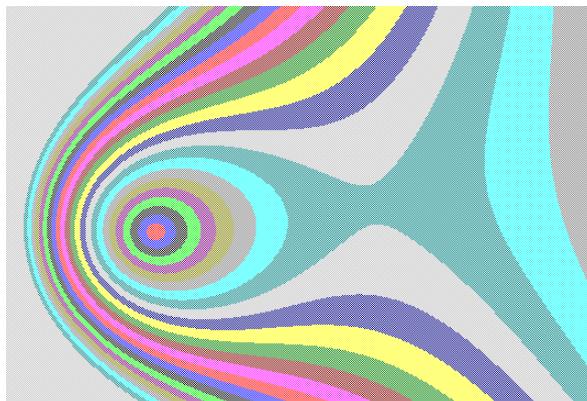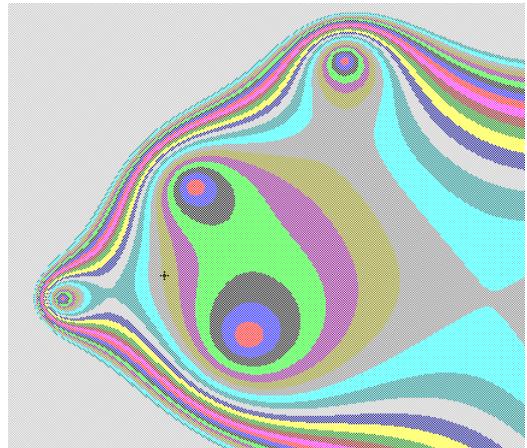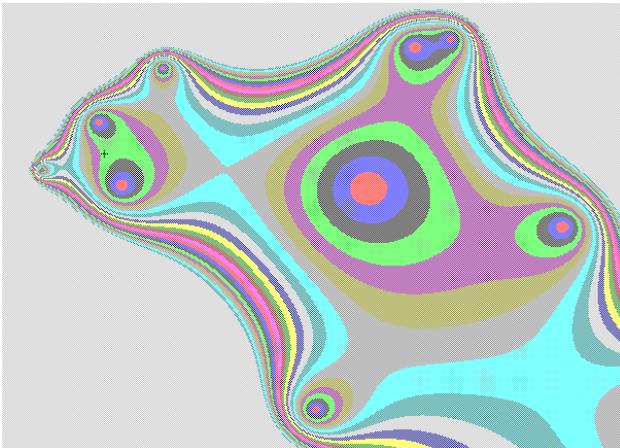
$$\text{julia}(z^2, -0.74543 + 0.11301 \cdot i)$$
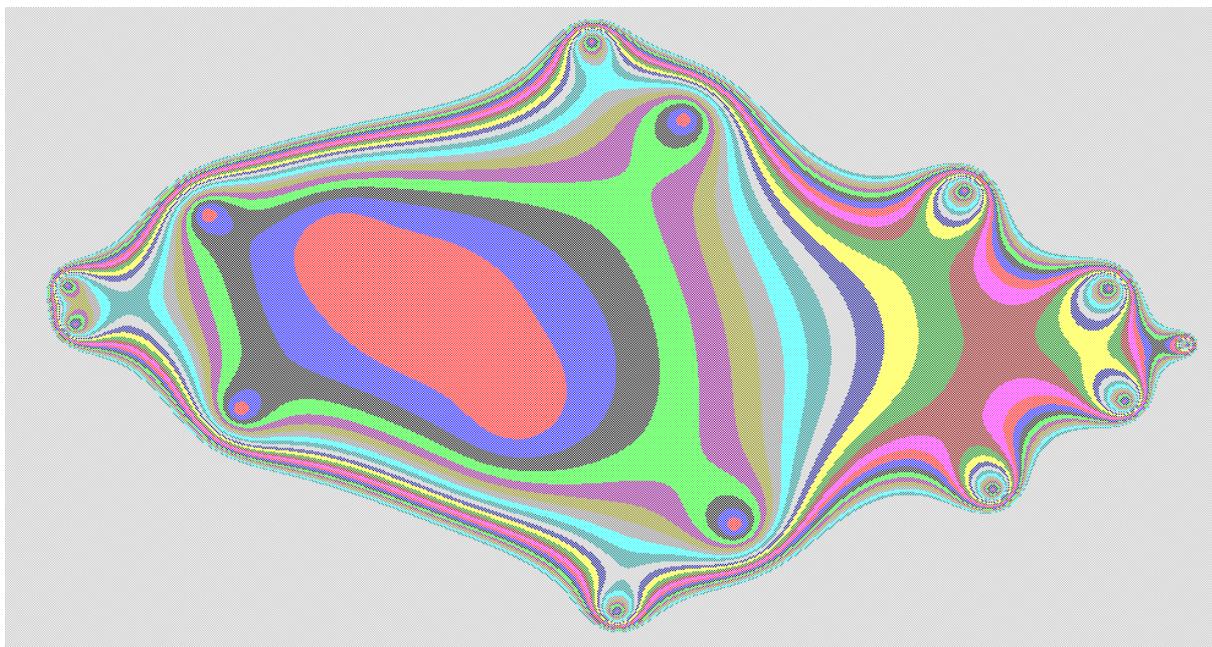
You can do stepwise plotting for $m = 0$ to $m = 3$ using steps of your choice. See the first two steps with $m = 1/20$. Three iterations seem to be sufficient, otherwise calculation times become too long or you might receive an Out of Memory message.

$$\left| \text{ITERATE}(3 \cdot z - 3 \cdot z^2, z, x + i \cdot y, 3) - x - i \cdot y \right| > 0 \land |x + i \cdot y| < 2$$

$$\left| \text{ITERATE}(3 \cdot z - 3 \cdot z^2, z, x + i \cdot y, 3) - x - i \cdot y \right| > \frac{1}{20} \land |x + i \cdot y| < 2$$
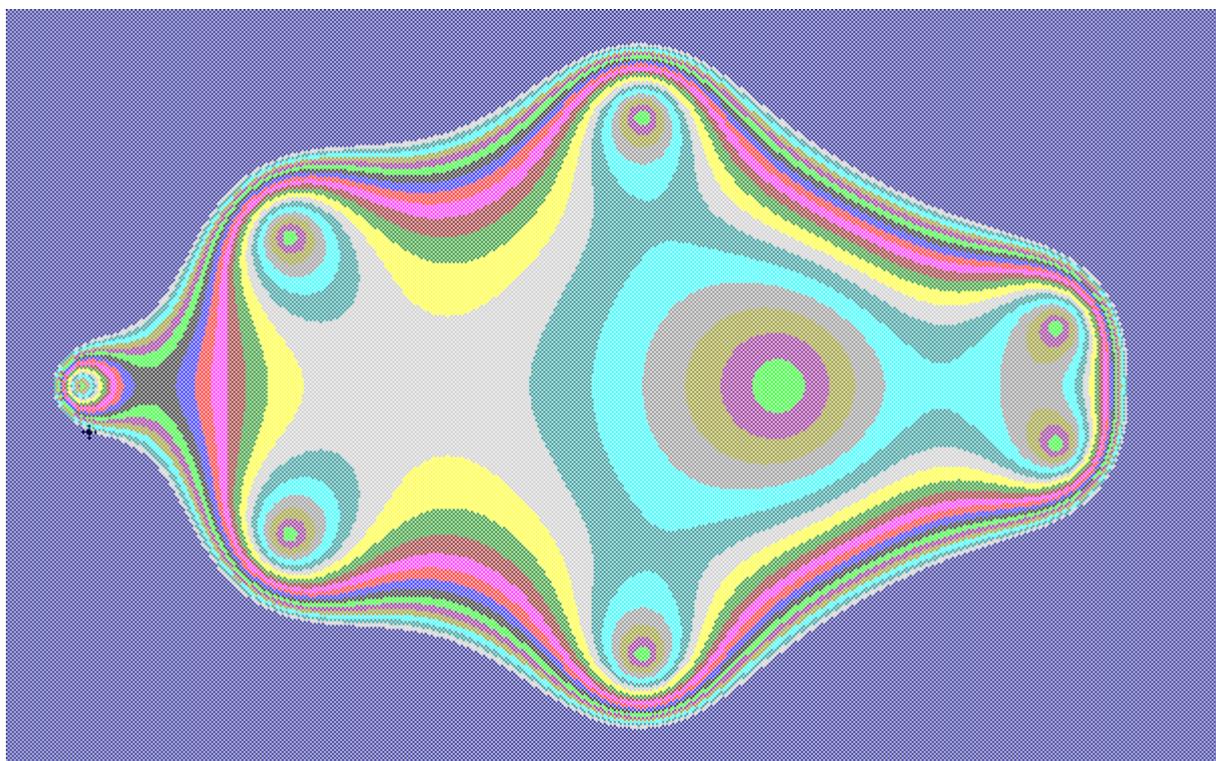
$$\left| \text{ITERATE}\left(z + \frac{0.1}{z^2}, z, x + i \cdot y, 3 \right) - x - i \cdot y \right| > 0 \land |x + i \cdot y| < 2$$
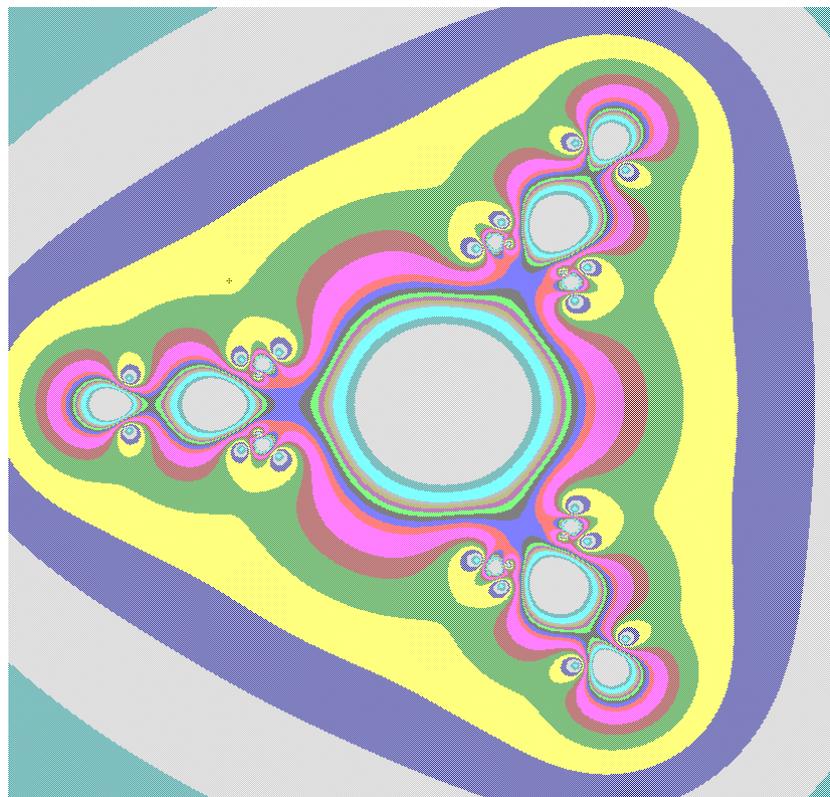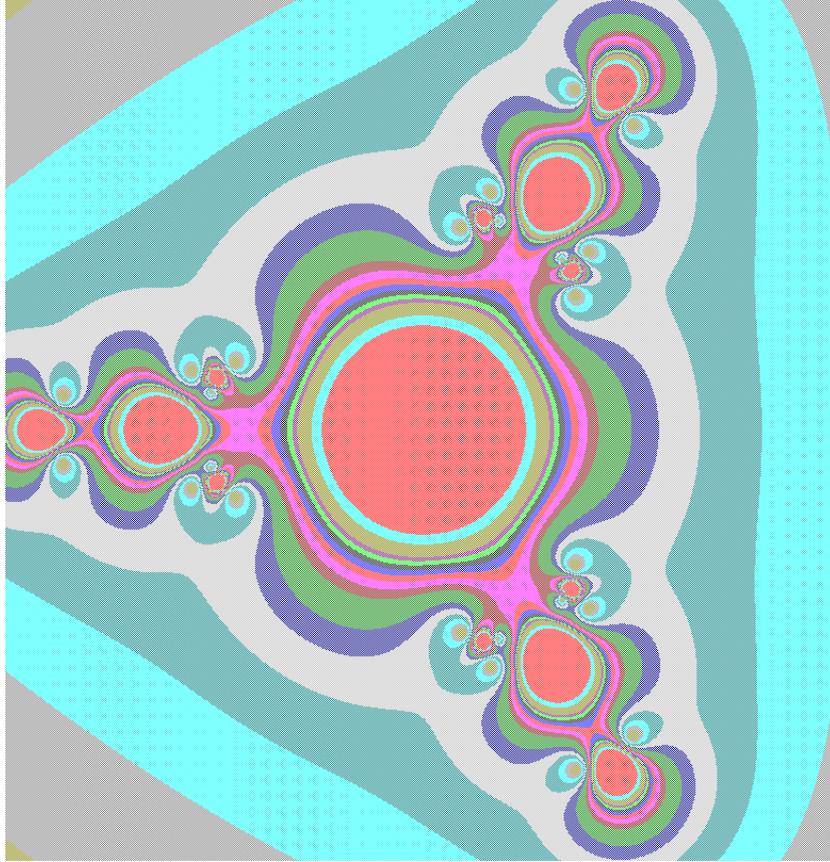

Two plots of the same figure with a different order of colors.

$$\text{julia}\left(z - \frac{0.1}{z},\ 0\right)$$
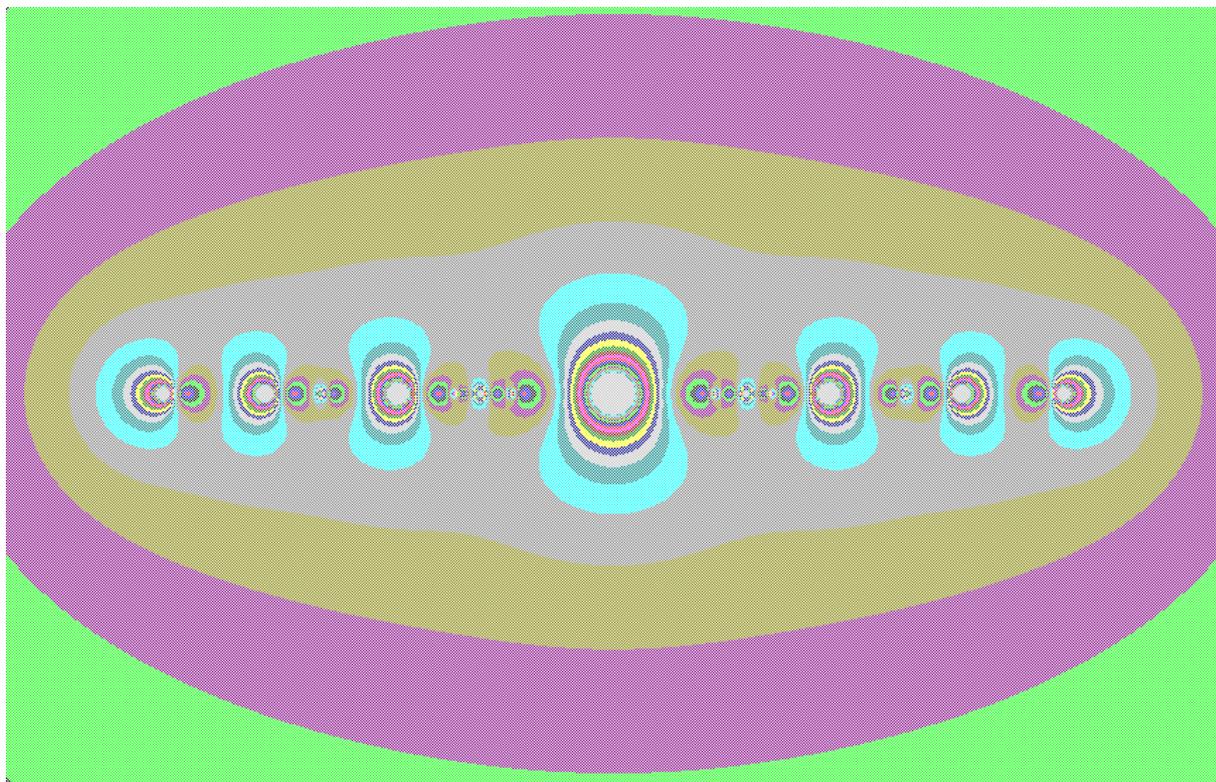
If we zoom into the centre of the figure, we get the following:
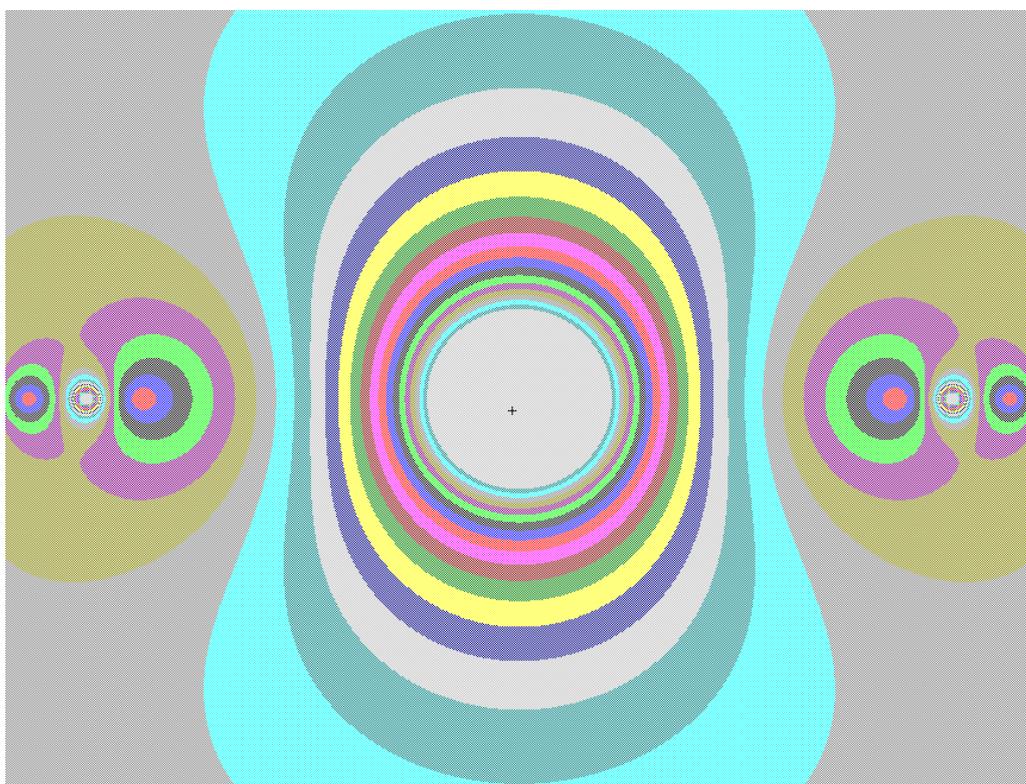
By changing one sign of the function, the following is the result:

$$\text{julia}\left(z + \frac{0.1}{z}, \ 0\right)$$
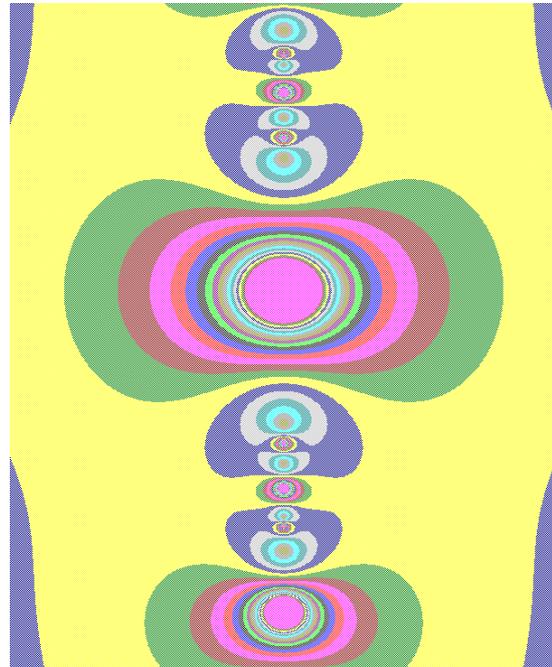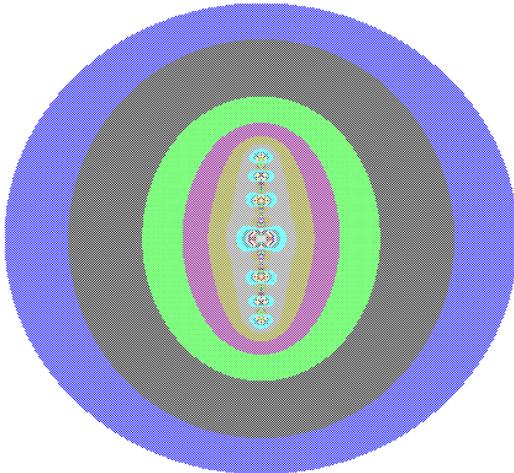
$$\left|\text{ITERATE}\left(z + \frac{0.2 + 0.4 \cdot i}{z^2}, \ z, \ x + i \cdot y, \ 3\right) - x - i \cdot y\right| > 0 \ \land \ |x + i \cdot y| < 2$$

...

$$\left|\text{ITERATE}\left(z + \frac{0.2 + 0.4 \cdot i}{z^2}, \ z, \ x + i \cdot y, \ 3\right) - x - i \cdot y\right| > 3 \ \land \ |x + i \cdot y| < 2$$

$$\frac{z^2 + 0.2 + 0.4 \cdot i}{z^2}$$

$$\frac{z^2 + 0.2 + 0.4 \cdot i}{z^2 + 1}$$
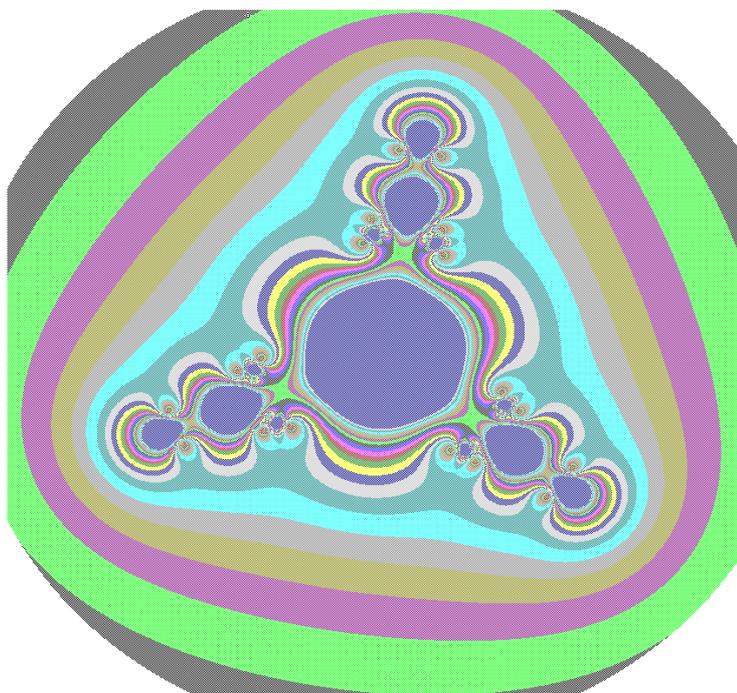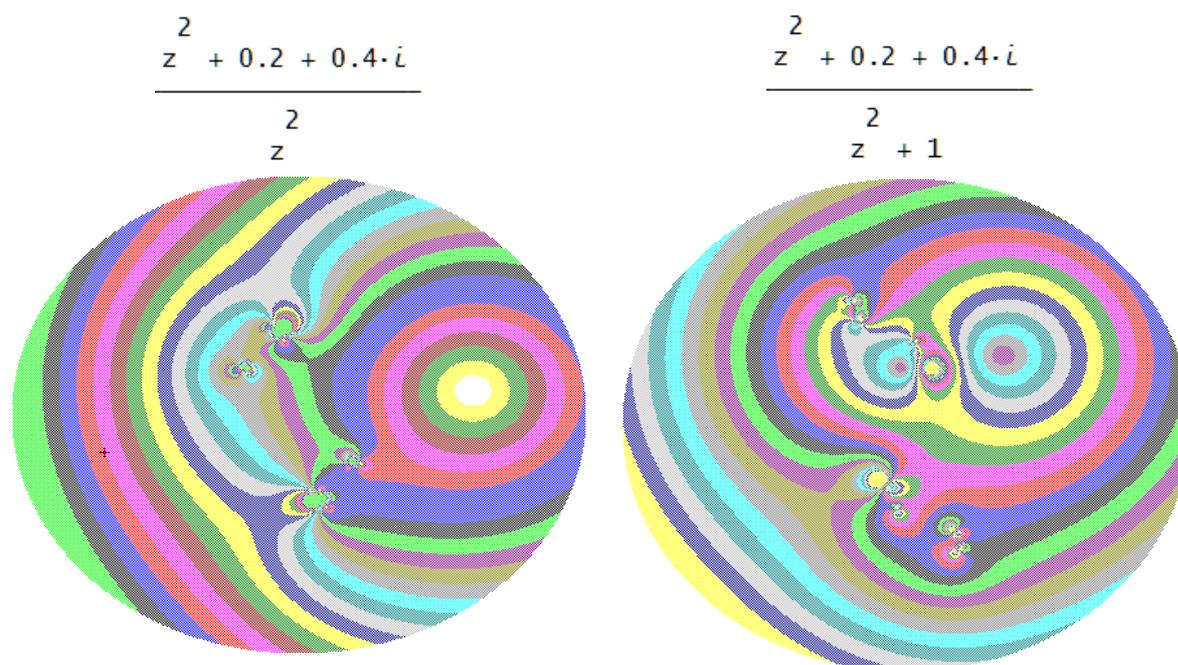
Warning: Plotting can take some time depending on the iteration function.

This is Roman's next example:

Example 26.1: Newton's iteration method

Display in the Gaussian plane of complex numbers the regions of attraction of each of the roots of the equation $z^3 - 1 = 0$, solving it based on Newton's iterative method.

Solution:

The sequence of solutions of the equation is given by the recurrence relation $x_{n+1} = x_n - \dfrac{g(x_n)}{g'(x_n)}$, where

$g(x) = x^3 - 1$. This can be rewritten as $x_{n+1} = f(x_n) = \dfrac{2x_n^3 + 1}{3x_n^2}$. Firstly, this function is defined, (see the

function for `julia` on page 4).

The `VECTOR` is then applied, the output of which is the vector of results comparing the value of $|f'(z)|$ with successively changing values of the parameter m. In doing so, we restrict ourselves to the region defined by the condition $|z| < 3$.

#1: $g(x) := x^3 - 1$

#2: $\left(f(z) := z - \dfrac{g(z)}{g'(z)} \right) = f(z) := \dfrac{2 \cdot z^3 + 1}{3 \cdot z^2}$

#3: $\text{VECTOR}\left(|f'(x + i \cdot y)| \leq m \wedge |x + i \cdot y| < 3,\ m,\ \dfrac{33}{8},\ \dfrac{1}{8},\ -\dfrac{1}{8} \right)$

The components of the resulting vector are relations of variables x, y. The graphical representation of each of them is an area colored with the same color (not exactly, when there are more areas than colors, the sequence of colors starts to repeat). Regions of the same color in the plane therefore correspond to points that approach the roots of the equation with approximately the same willingness.
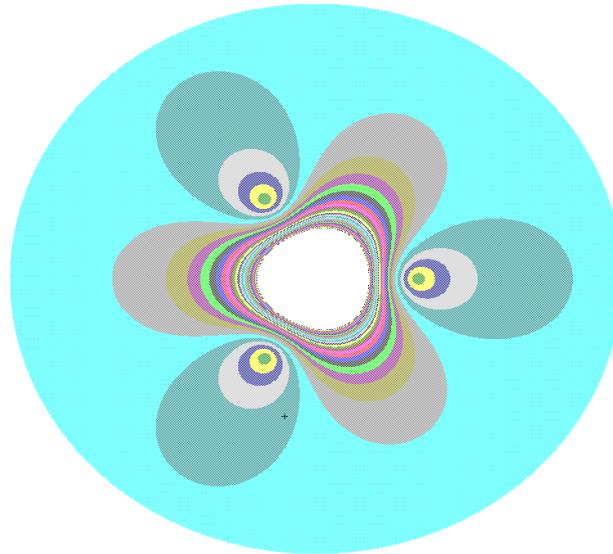
This was Roman's explication and his sample graph (in book, gray scale only).

I could not resist experimenting with Roman's function and produced a gallery of Newton-pictures.

$$g(x) := x^3 + x^2 + x$$

$$\left(f(z) := z - \frac{g(z)}{g'(z)} \right) = f(z) := \frac{z^2 \cdot (2 \cdot z + 1)}{3 \cdot z^2 + 2 \cdot z + 1}$$

$$\text{VECTOR}\left(\left| f'(x + i \cdot y) \right| \le m \land \left| x + i \cdot y \right| < 3, \ m, \ \frac{33}{8}, \ \frac{1}{8}, \ -\frac{1}{8} \right)$$
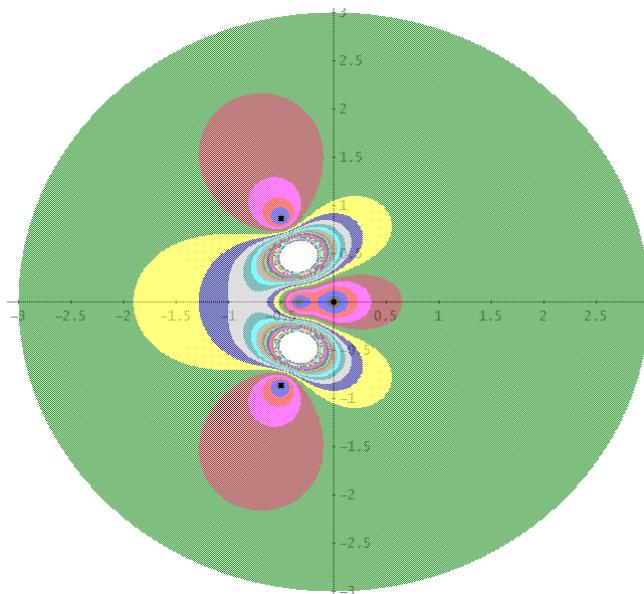
What happens if we apply the VECTOR command on g(x)?

Beware: This is **not** the Newton method!!

$$\text{VECTOR}\left(\left|g'(x + i \cdot y)\right| \le m \wedge \left|x + i \cdot y\right| < 2,\ m,\ \frac{33}{8},\ \frac{1}{8},\ -\frac{1}{8}\right)$$
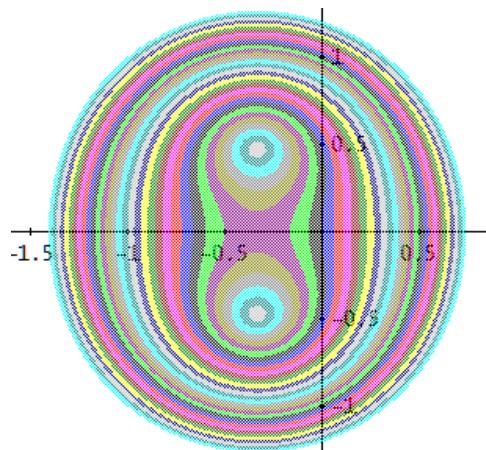
$$g(x) := x^4 + 4 \cdot x^2 + 3$$

$$\left(f(z) := z - \frac{g(z)}{g'(z)}\right) = f(z) := \frac{3 \cdot z^4 + 4 \cdot z^2 - 3}{4 \cdot z \cdot (z^2 + 2)}$$
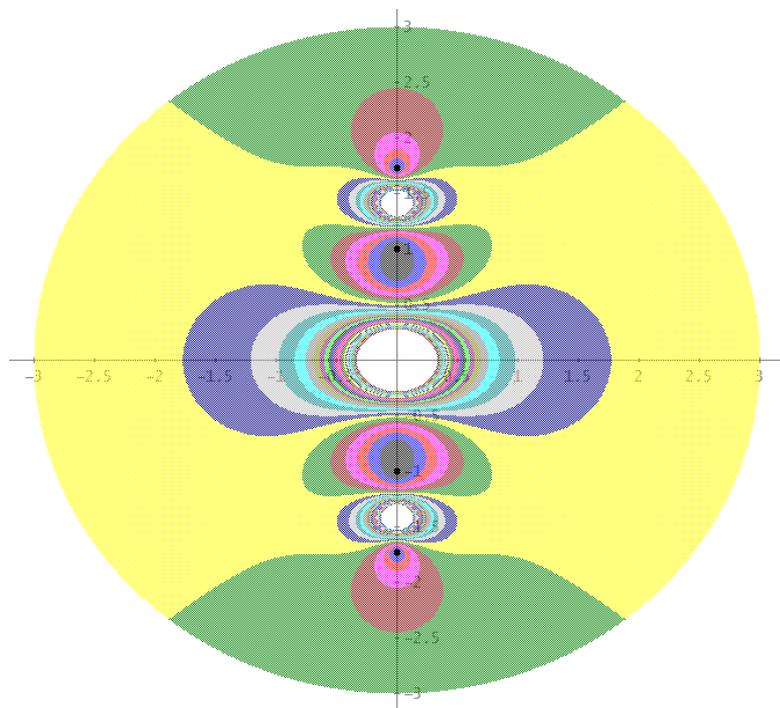
$$g(x) := x^4 + 16 + 6 \cdot i \cdot x^2$$

$$\left(f(z) := z - \frac{g(z)}{g'(z)}\right) = f(z) := -\frac{25 \cdot z}{4 \cdot (z^4 + 9)} + \frac{3 \cdot z}{4} + \frac{3 \cdot i \cdot (16 - z^4)}{4 \cdot z \cdot (z^4 + 9)}$$
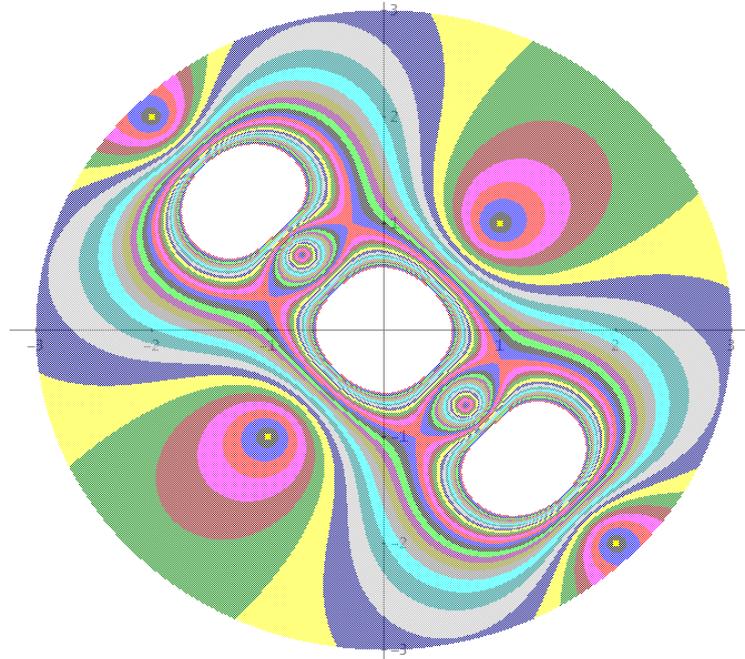
The yellow points are the solution points of the equation.

The procedure is always the same, therefore only the function $g(x)$ is shown. Either plot using the Newton method (creating $f(z)$) or use $g(x)$ in the VECTOR command.

$$g(x) := \frac{3 \cdot x^4 + 1}{3 \cdot x^2}$$
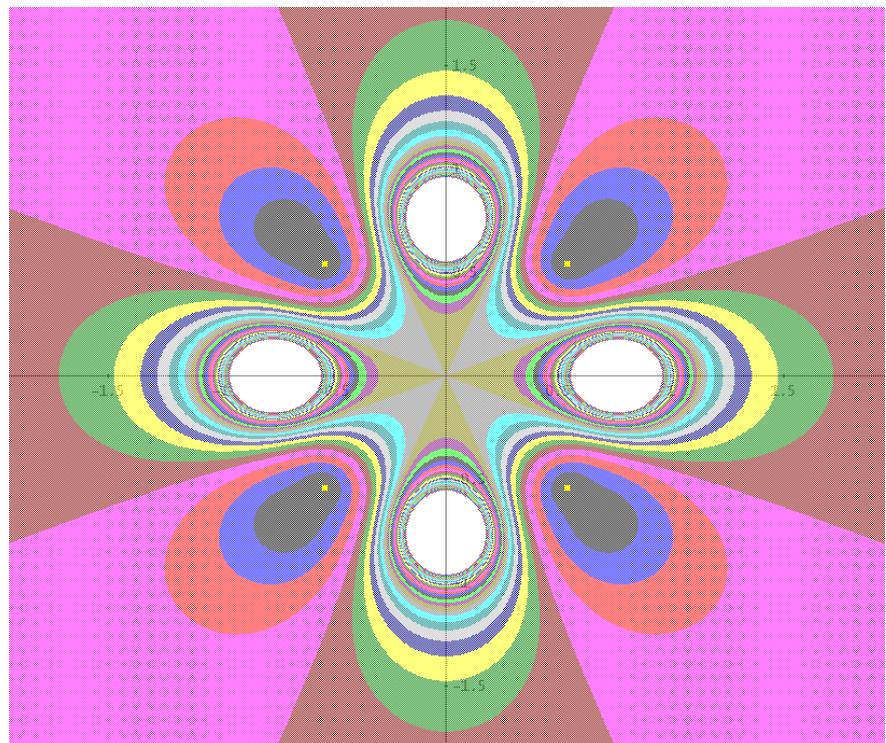
Compare with:

$$\text{VECTOR}\left(|g'(x + i \cdot y)| \le m \wedge |x + i \cdot y| < 3, \ m, \ \frac{33}{8}, \ \frac{1}{8}, \ -\frac{1}{8} \right)$$
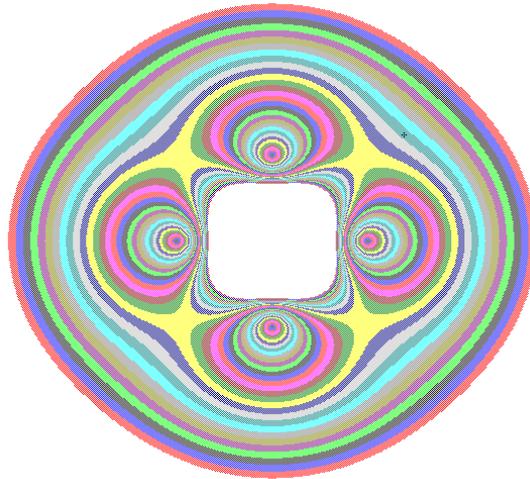
Same procedure for

$$g(x) := \frac{3 \cdot x^4 + x - 2}{3 \cdot x^2 - 1}$$
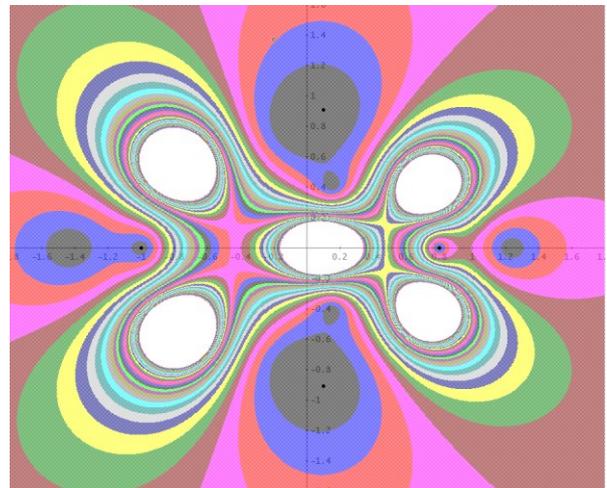
Followed by

$$\text{VECTOR}\left(\left|g'(x + i \cdot y)\right| \le m \wedge \left|x + i \cdot y\right| < 3,\ m,\ \frac{33}{8},\ \frac{1}{8},\ -\frac{1}{8}\right)$$
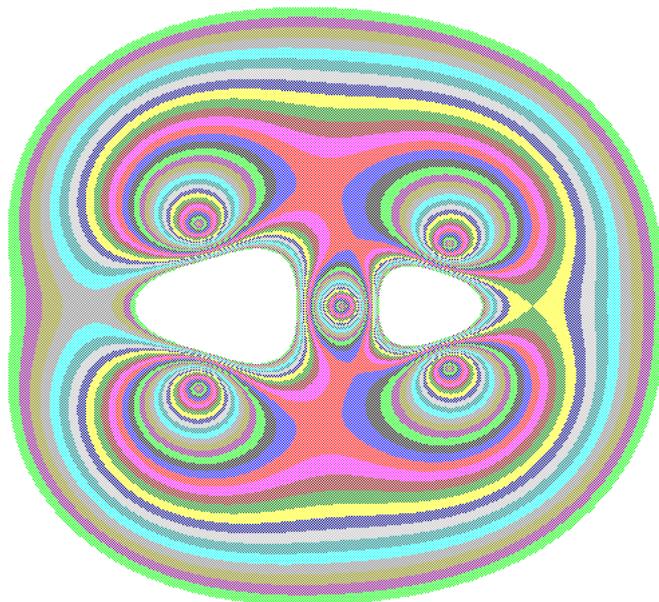
$$g(x) := \frac{3 \cdot x^4 + 4 \cdot x^2 - 3}{4 \cdot x \cdot (x^2 + 2)}$$
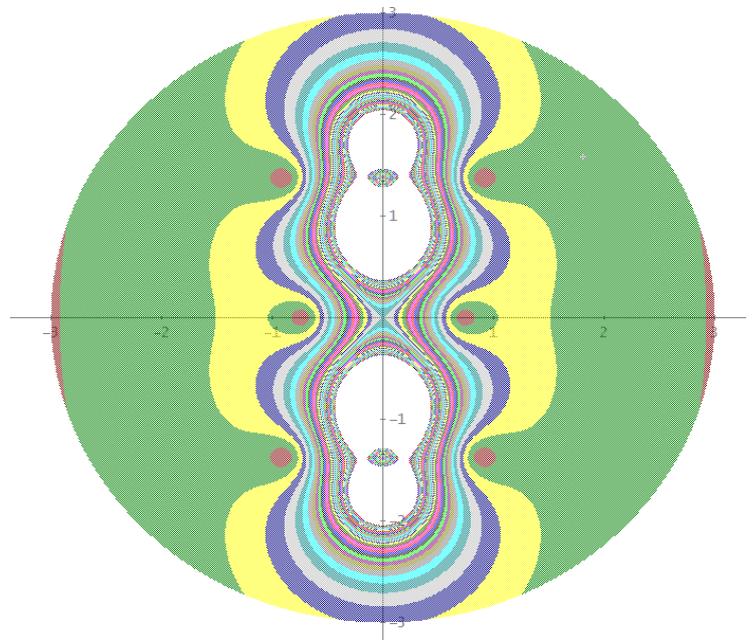
If we zoom in:
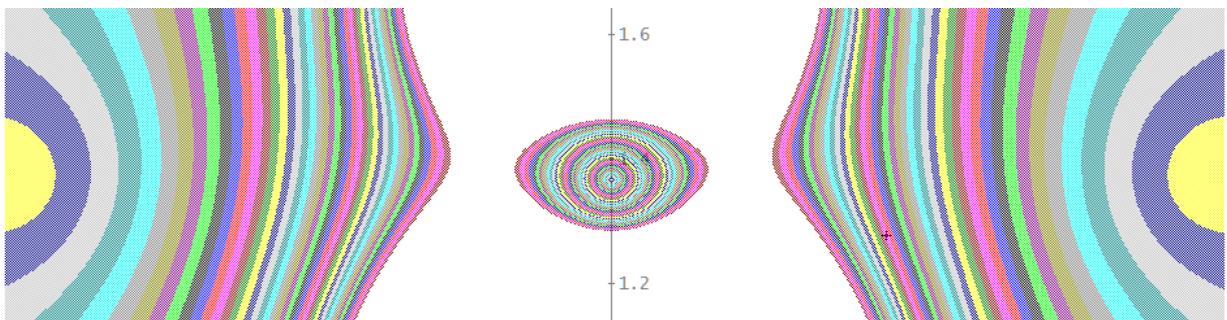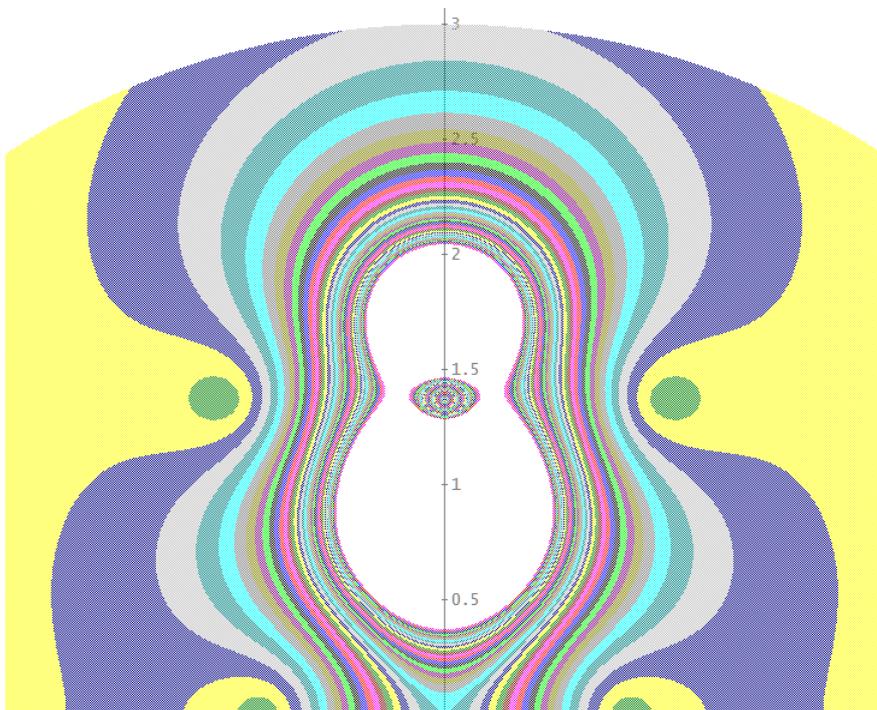
$$g(x) := x^5 - 2 \cdot x^4 + 3 \cdot x^3 - 4 \cdot x^2 + x - 1$$
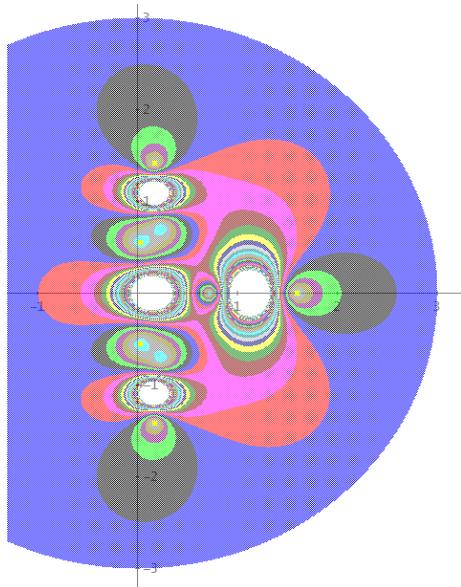
Newton   Not Newton
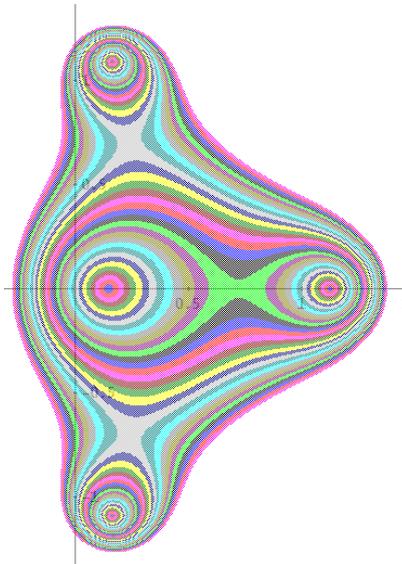
$$g(x) := SIN(x) + COS(x)$$

$$\left(f(z) := z - \frac{g(z)}{g'(z)} \right) = f(z) := \frac{(z-1) \cdot COS(z) - (z+1) \cdot SIN(z)}{COS(z) - SIN(z)}$$

$$VECTOR\left(|f'(x + i \cdot y)| \leq m \wedge |x + i \cdot y| < 3, \ m, \ \frac{33}{8}, \ \frac{1}{8}, \ -\frac{1}{8} \right)$$
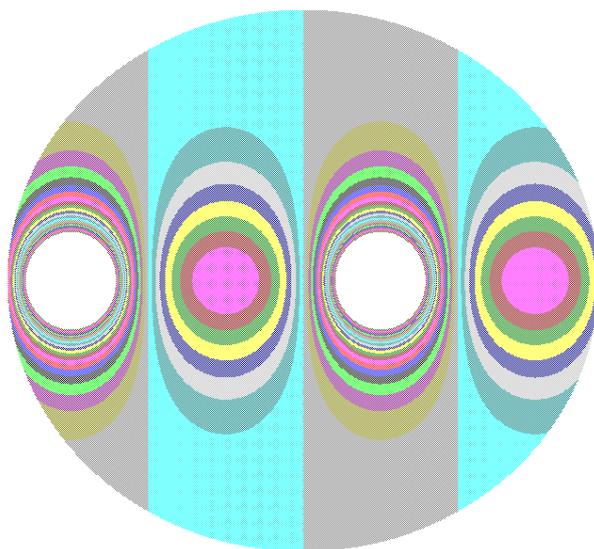
Beware: Plotting is sometimes time consuming!

[1] Hašek, R. *Užití Derive ve výuce matematiky*. České Budějovice: Europeon, a. s., 2007

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https://compasstech.com.au/

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  Preview ⇒

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  Preview ⇒

- Magnetometer and GPS Navigation from the STEM HQ (Choose another?)
  Preview ⇒

- ProbSim: Probability Simulations from the Lua and Javascript Showcase
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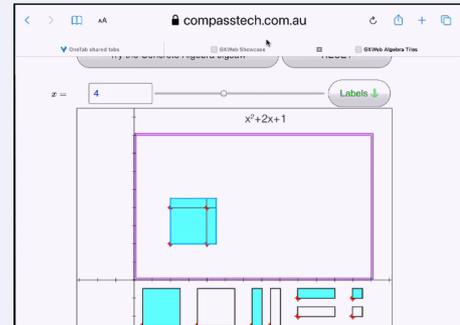
Algebra Tiles Jigsaw from the
GXWeb Jigsaws and Quizzes
(*Tap the image to switch views!*)

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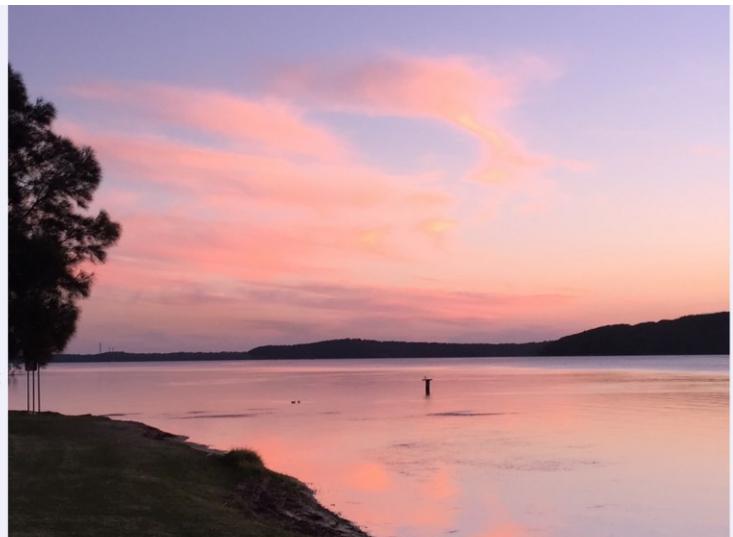
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W3Schools.com: Taking Your Web Page Skills to the Next Level

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GXWeb Polygon Playground 1

Saltire Software, home of *Geometry Expressions* and *GXWeb*

Online Encyclopedia of Integer Sequences (A357442): Explore polygon diagonals further in this extensive collection of information directly related to Challenge 2 of our Polygon Playground!

GXWeb Polygon Playground 2: Angle Theorems

GXWeb Polygon Playground: YouTube Introduction (35:20)

- Introduction

- Games 1 and 2: Dealing with Diagonals

- GXWeb Polygon Model

- How many ways did you say?

- Game 3: GXWeb Polygonal Angles Theorem Generator

- Game 4: GXWeb Cyclic Polygon Constructions

- Build Your Own Model with GXWeb

- Share Your Results

- Behind the Scenes

https://compasstech.com.au/gxwconstruct/index.html
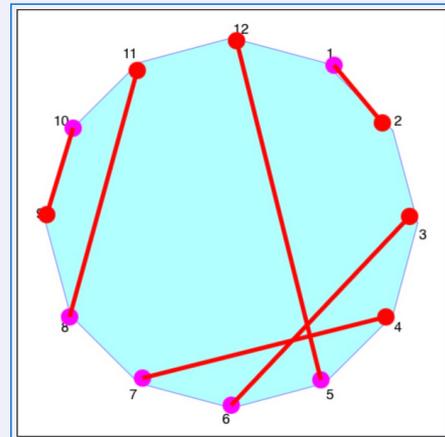
Mechanisms for Constructing Plane Curves

This is a sampler of a much more extensive YouTube PlayList by Phil Todd of *Saltire Software*, creator of such wonderful mathematics learning and exploration tools as *Geometry Expressions, Mathematical Illustrations* and the amazing, free browser-based GXWeb, which is the focus here.

Phil uses GXWeb to construct curves as outlined in the book, **Mechanisms for the Construction of Plane Curves** by *I. I. Artobolevskii*
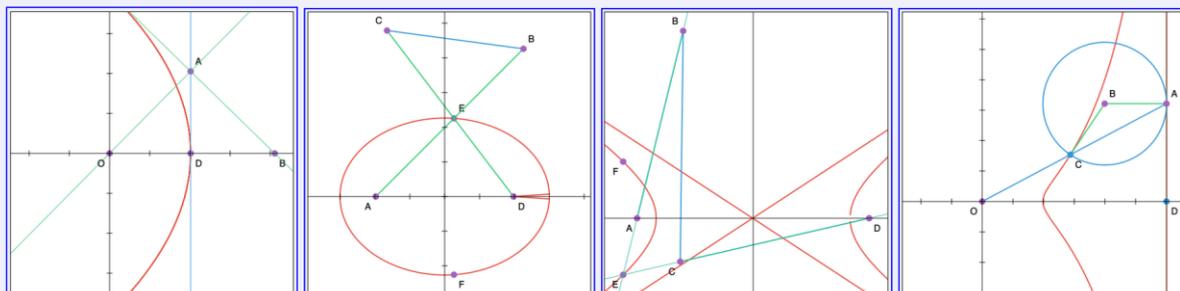
| Enveloped Parabola | Ellipse | Hyperbola | Cissoid of Diocles |
| View on YouTube | View on YouTube | View on YouTube | View on YouTube |

| Lemniscate of Bernoulli | Pedal Curves | Lemniscatoid | Witch of Agnesi |
| View on YouTube | View on YouTube | View on YouTube | View on YouTube |
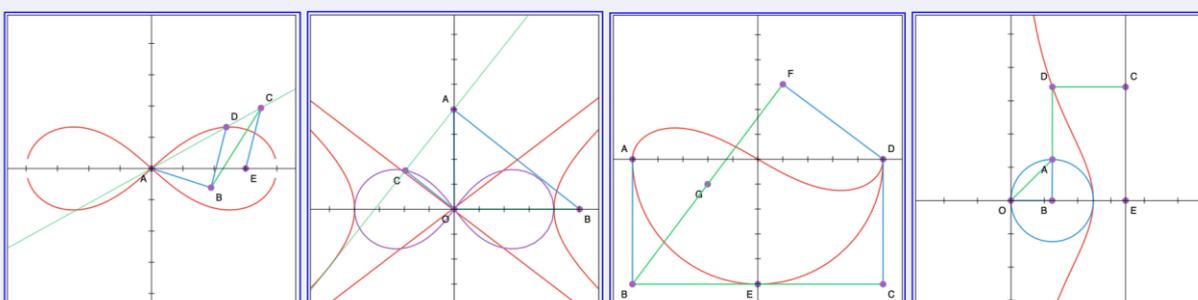
Heinrich Hemme 3

Wolfgang Alvermann and Josef Böhm

Heinrich Hemme publishes regularly mathematical puzzles on www.spektrum.de. Very often they have been inspirations for Wolfgang Alvermann not only to solve them, but to prepare them together with additional tasks for his colleagues (and for students, of course, too).

Wolfgang prefers – in almost all cases – traditional calculations with paper and pen. I like to play around with CAS and sometimes with dynamic geometry.

What is the Radius?

https://www.spektrum.de/raetsel/welchen-radius-hat-der-kreis/2002909

"A heart composed of a square with side length 1 and two semicircles is enclosed in a circle.
What is the radius r of the circle?"
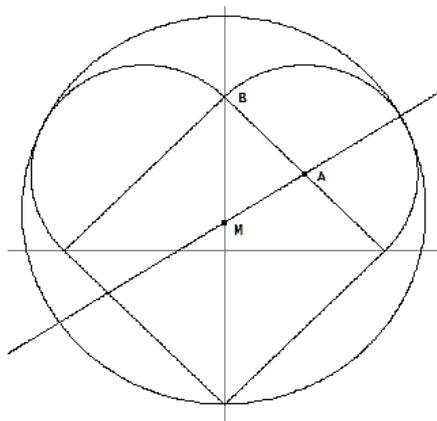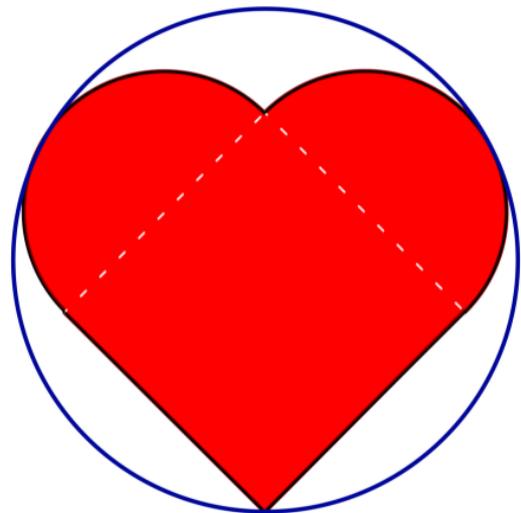
Applying the cosine rule in triangle MAB I calculate distance MA, expressing all sides using side length of the square (= a) and the radius r of the circumcircle.

The equation can be solved for r. It turns out to be a linear equation. CAS is helpful – but not really necessary – see Wolfgang's solution below.

I simulate the manual calculation:

#1: $\left(\dfrac{a}{2}\right)^2 + (a\cdot\sqrt{2} - r)^2 - a\cdot(a\cdot\sqrt{2} - r)\cdot\cos\left(\dfrac{\pi}{4}\right) = \left(r - \dfrac{a}{2}\right)^2$

#2: $\dfrac{5\cdot a^2 - 6\cdot\sqrt{2}\cdot a\cdot r + 4\cdot r^2}{4} = \dfrac{(a - 2\cdot r)^2}{4}$

#3: $\dfrac{5\cdot a^2 - 6\cdot\sqrt{2}\cdot a\cdot r + 4\cdot r^2}{4} - \dfrac{(a - 2\cdot r)^2}{4} = 0$

#4: $a\cdot(\sqrt{2}\cdot a + r\cdot(\sqrt{2} - 3)) = 0$

#5: $\text{SOLVE}(a\cdot(\sqrt{2}\cdot a + r\cdot(\sqrt{2} - 3)) = 0, r)$

#6: $r = a\cdot\left(\dfrac{3\cdot\sqrt{2}}{7} + \dfrac{2}{7}\right) \ \lor \ a = 0$

It is much more elegant using only one single line:

$\text{SOLVE}\left(\left(\dfrac{a}{2}\right)^2 + (a\cdot\sqrt{2} - r)^2 - a\cdot(a\cdot\sqrt{2} - r)\cdot\cos\left(\dfrac{\pi}{4}\right) = \left(r - \dfrac{a}{2}\right)^2, r\right)$

$r = a\cdot\left(\dfrac{3\cdot\sqrt{2}}{7} + \dfrac{2}{7}\right) \ \lor \ a = 0$

I found the radius of the circumcircle and can plot heart + circumcircle for arbitrary values of *a*.
Then I had an idea to extend the problem. But see Wolfgang's solution first.

Wolfgang prefers – if possible – traditional calculations without using CAS. Shame on me that I always act as a "CASer". I should also try to come back to the roots. See here how he tackled the problem:

Welchen Radius hat der Kreis?

Solution:

The following relationships can be read off from the extended drawing:

$r = x + \dfrac{1}{2}$

$x = r - \dfrac{1}{2}$

And applying the cosine rule in triangle MAB:

$x^2 = \left(\dfrac{1}{2}\right)^2 + \left(\sqrt{2} - r\right)^2 - 2\cdot\dfrac{1}{2}\cdot\left(\sqrt{2} - r\right)\cdot\cos(45°)$

$x^2 = \dfrac{1}{4} + 2 - 2\cdot\sqrt{2}\cdot r + r^2 - 1 + \dfrac{1}{2}\cdot\sqrt{2}\cdot r$

With $x^2 = r^2 - r + \dfrac{1}{4}$ follows:
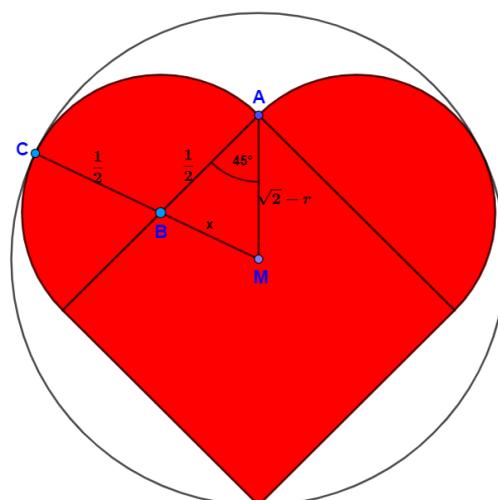
$$\cancel{r^2} - r + \cancel{\frac{1}{4}} = \cancel{\frac{1}{4}} + 2 - 2\cdot\sqrt{2}\cdot r + \cancel{r^2} - 1 + \frac{1}{2}\cdot\sqrt{2}\cdot r \;\bigg| \quad \frac{3}{2}\cdot\sqrt{2}\cdot r - r = 1$$

$$-r = 1 - \frac{3}{2}\cdot\sqrt{2}r \quad\bigg\} \;\rightarrow\; r = \frac{2}{3\cdot\sqrt{2} - 2}$$

Making the denominator rational one gets finally:

$$\boxed{\begin{array}{l} r = \dfrac{3\cdot\sqrt{2} + 2}{7} \\[4pt] r \approx 0.891806 \end{array}}$$

My excuse for thinking as a "CASer" is that I wanted to plot a filled (bloody) heart and circumscribed circle with variable *a*. I can imagine that this might be a nice problem for the students:

#1: $\;\; y \le \dfrac{a\cdot\sqrt{2} - 2\cdot x}{2} \;\land\; y \le \dfrac{-a\cdot\sqrt{2} + 2\cdot x}{2} \;\land\; y \le \dfrac{-a\cdot\sqrt{2} - 2\cdot x}{2} \;\land\; y \le \dfrac{a\cdot\sqrt{2} + 2\cdot x}{2}$

#2: $\;\; x^2 + \left(y + \dfrac{a\cdot\sqrt{2}}{2} - \dfrac{a\cdot(3\cdot\sqrt{2} + 2)}{7}\right)^2 = \left(\dfrac{a\cdot(3\cdot\sqrt{2} + 2)}{7}\right)^2$

#3: $\;\; \mathrm{sq}(a) := y \ge |x| - \dfrac{a\cdot\sqrt{2}}{2} \;\land\; y \le -|x| + \dfrac{a\cdot\sqrt{2}}{2}$

#4: $\;\; \mathrm{cs}(a) := \left(x - \dfrac{a\cdot\sqrt{2}}{4}\right)^2 + \left(y - \dfrac{a\cdot\sqrt{2}}{4}\right)^2 \le \left(\dfrac{a}{2}\right)^2 \;\lor\; \left(x + \dfrac{a\cdot\sqrt{2}}{4}\right)^2 + \left(y - \dfrac{a\cdot\sqrt{2}}{4}\right)^2 \le \left(\dfrac{a}{2}\right)^2$

#5: $\;\; \mathrm{heart}(a) := \mathrm{sq}(a) \lor \mathrm{cs}(a)$

#16: $\;\; \mathrm{circle}(a) := x^2 + \left(y + \dfrac{a\cdot\sqrt{2}}{2} - \dfrac{a\cdot(3\cdot\sqrt{2} + 2)}{7}\right)^2 = \left(\dfrac{a\cdot(3\cdot\sqrt{2} + 2)}{7}\right)^2$

#17: $\;\; \mathrm{heart}(2)$

#18: $\;\; \mathrm{circle}(2)$
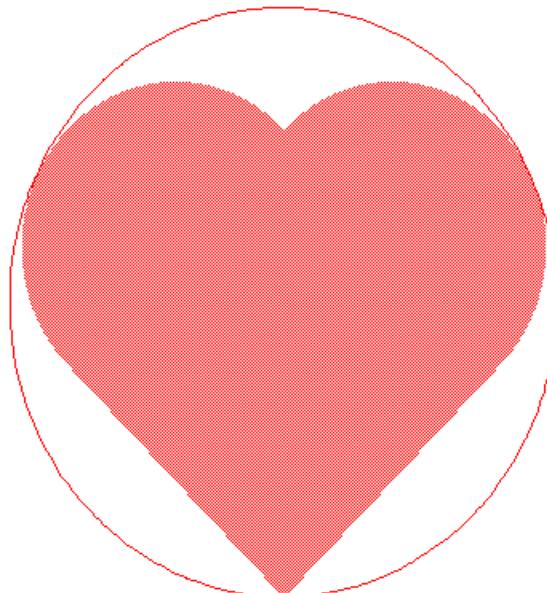
The second puzzle is the same question, but another problem:

What is the Radius?

https://www.spektrum.de/raetsel/wie-gross-ist-der-radius/2156913
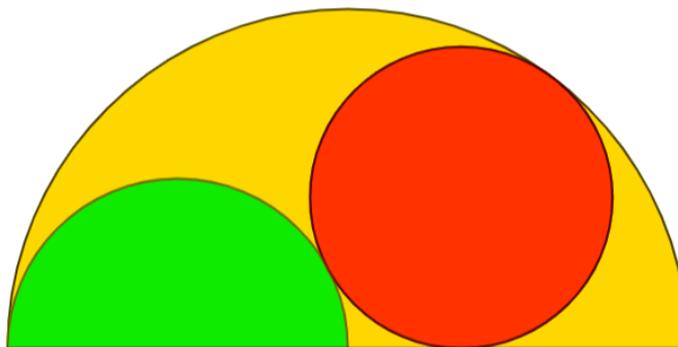
"A semicircle of radius 18 contains a semicircle of radius 9
and a full circle. What is the radius of the full circle?"

The circle wanted (the red one) has its center in the intersection point of the circles k_1 and k_2:

```
#1:    CaseMode := Sensitive

       The given semicircles (black): :
```

$$\text{\#2:} \quad y = \sqrt{(81 - (x + 9)^2)}$$

$$\text{\#3:} \quad y = \sqrt{(18^2 - x^2)}$$

```
       k1 in blue and k2 in red
```

$$\text{\#4:} \quad k1 := (x + 9)^2 + y^2 = (9 + r)^2$$

$$\text{\#5:} \quad k2 := x^2 + y^2 = (18 - r)^2$$

```
#6:    SOLUTIONS(k1 ∧ k2, [x, y])
```

$$\text{\#7:} \quad \begin{bmatrix} 3 \cdot r - 18 & 2 \cdot \sqrt{2} \cdot \sqrt{(r \cdot (9 - r))} \\ 3 \cdot r - 18 & - 2 \cdot \sqrt{2} \cdot \sqrt{(r \cdot (9 - r))} \end{bmatrix}$$

```
       I take the intersection point with posive y-value
       and then I plot circles with variable radius r using
       a slider for r (black):
```

$$\text{\#8:} \quad [3 \cdot r - 18, \; 2 \cdot \sqrt{2} \cdot \sqrt{(r \cdot (9 - r))}]$$

$$\text{\#9:} \quad (x - 3 \cdot r + 18)^2 + (y - 2 \cdot \sqrt{2} \cdot \sqrt{(r \cdot (9 - r))})^2 = r^2$$

I show three positions of the circle with different *r*: Plots for DERIVE (left) and TI-Nspire (right).

$$(x-3\cdot\mathbf{r}+18)^2+\left(y-2\cdot\sqrt{18\cdot\mathbf{r}-2\cdot\mathbf{r}^2}\right)^2=\mathbf{r}^2$$

$$(x-3\cdot\mathbf{r}+18)^2+\left(y-2\cdot\sqrt{18\cdot\mathbf{r}-2\cdot\mathbf{r}^2}\right)^2=\mathbf{r}^2$$

$$(x-3\cdot\mathbf{r}+18)^2+\left(y-2\cdot\sqrt{18\cdot\mathbf{r}-2\cdot\mathbf{r}^2}\right)^2=\mathbf{r}^2$$

$$(x-3\cdot\mathbf{r}+18)^2+\left(y-2\cdot\sqrt{18\cdot\mathbf{r}-2\cdot\mathbf{r}^2}\right)^2=\mathbf{r}^2$$
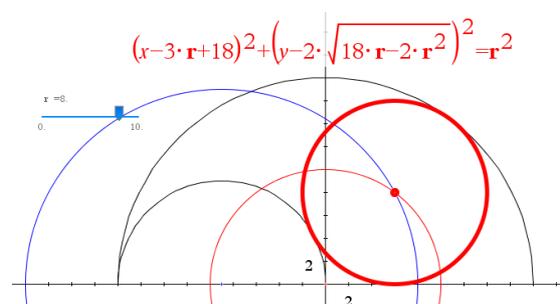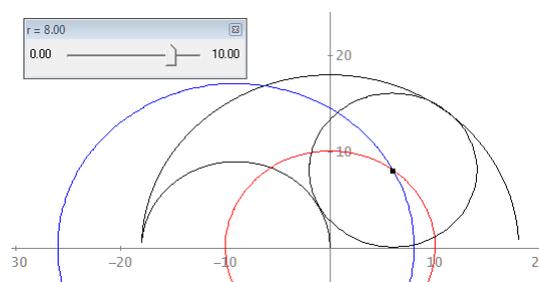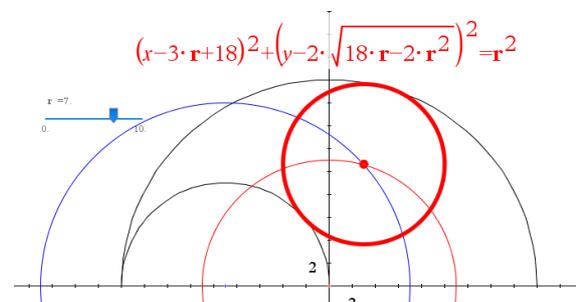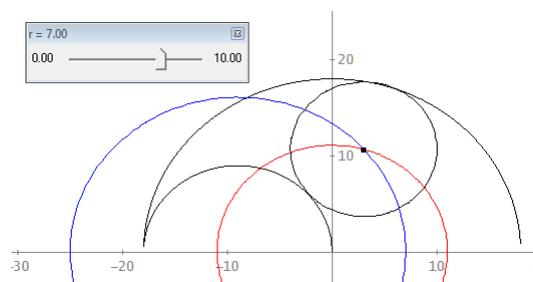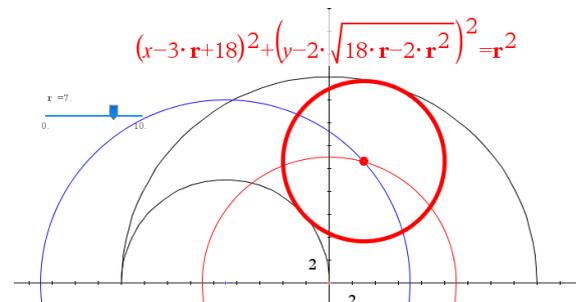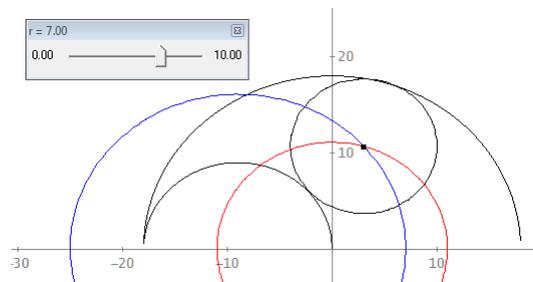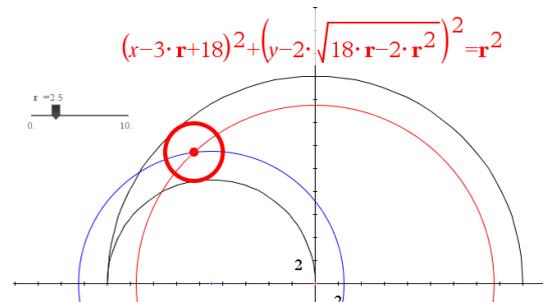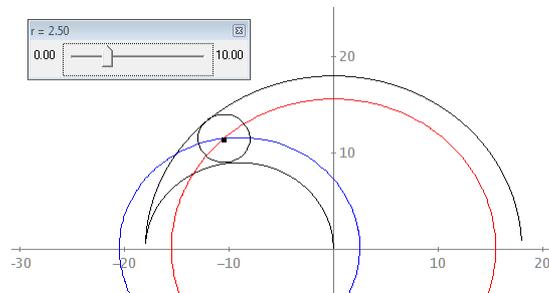
The suspicion is obvious that *r* = 8 is the solution.

The *y*-value of the center of the circle wanted must equal its radius, so:

#14: $y \le \sqrt{(18^2 - x^2)} \wedge y \ge 0$

#15: $y \le \sqrt{(81 - (x + 9)^2)} \wedge y \ge 0$

#16: $x^2 - 12\cdot x + y^2 - 16\cdot y + 100 \le 64$

Now we can plot the solution:
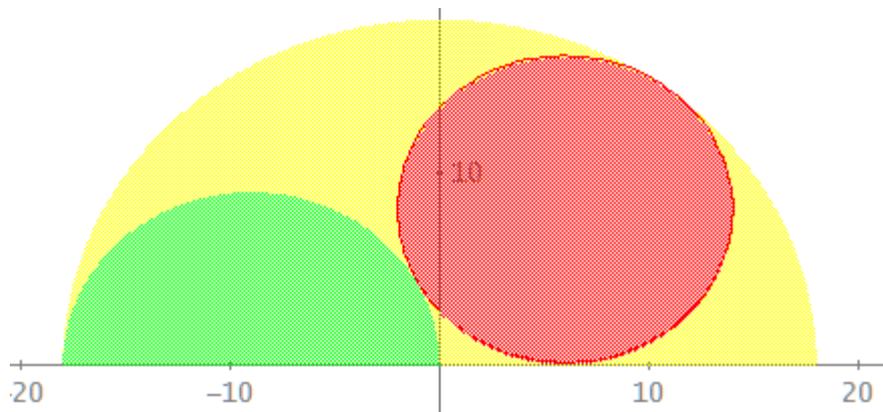
The problem can be generalized: The large circle has radius R, the small one has radius ρ:

#17: $k3 := (x - \rho)^2 + y^2 = (\rho + r)^2$

#18: $k4 := x^2 + y^2 = (R - r)^2$

#19: SOLUTIONS(k3 ∧ k4, [x, y])

#20:
$$\left[\begin{array}{cc} \dfrac{R^2 - 2 \cdot R \cdot r - 2 \cdot r \cdot \rho}{2 \cdot \rho} & \dfrac{\sqrt{(R \cdot (R + 2 \cdot \rho) \cdot (R - 2 \cdot r - 2 \cdot \rho) \cdot (2 \cdot r - R))}}{2 \cdot |\rho|} \\ \dfrac{R^2 - 2 \cdot R \cdot r - 2 \cdot r \cdot \rho}{2 \cdot \rho} & -\dfrac{\sqrt{(R \cdot (R + 2 \cdot \rho) \cdot (R - 2 \cdot r - 2 \cdot \rho) \cdot (2 \cdot r - R))}}{2 \cdot |\rho|} \end{array}\right]$$

#21: $\text{SOLUTIONS}\left(\dfrac{\sqrt{(R \cdot (R + 2 \cdot \rho) \cdot (R - 2 \cdot r - 2 \cdot \rho) \cdot (2 \cdot r - R))}}{2 \cdot |\rho|} = r, \ r\right)$

#22:
$$\left[\begin{array}{c} \dfrac{R \cdot (2 \cdot \rho^{3/2} \cdot \sqrt{(R + 2 \cdot \rho)} + R^2 + R \cdot \rho - 2 \cdot \rho^2)}{2 \cdot (R^2 + 2 \cdot R \cdot \rho + \rho^2)}, \ - \\ \dfrac{R \cdot (2 \cdot \rho^{3/2} \cdot \sqrt{(R + 2 \cdot \rho)} - R^2 - R \cdot \rho + 2 \cdot \rho^2)}{2 \cdot (R^2 + 2 \cdot R \cdot \rho + \rho^2)} \end{array}\right]$$

#23: $\text{SUBST}\left(r = \dfrac{R \cdot (2 \cdot \rho^{3/2} \cdot \sqrt{(R + 2 \cdot \rho)} + R^2 + R \cdot \rho - 2 \cdot \rho^2)}{2 \cdot (R^2 + 2 \cdot R \cdot \rho + \rho^2)}, \ [R, \rho], \ [18, 9]\right)$

#24: `r = 8`

Wolfgang's solution is much shorter (as always - shame on me!)

Wie groß ist der Radius?

https://www.spektrum.de/raetsel/wie-gross-ist-der-radius/2156913

Solution:

Triangle ACD:

$$(9+r)^2 = (9+x)^2 + r^2$$

Triangle BCD:

$$(18-r)^2 = r^2 + x^2$$
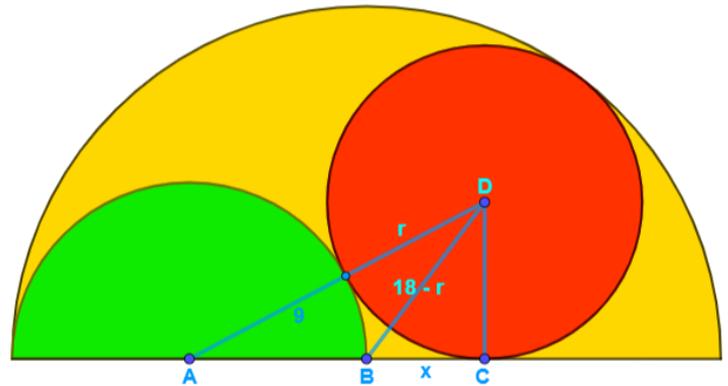
This results in the following two equations:

$$\left.\begin{array}{l} 81+18r+r^2 = 81+18x+x^2+r^2 \\ 324-36x+r^2 = r^2+x^2 \end{array}\right\} \rightarrow \left.\begin{array}{l} 36r = 36x+2x^2 \\ 36r = 324-x^2 \end{array}\right\} \rightarrow 3x^2+36x-324 = 0$$

The quadratic $x^2 + 12x - 108 = 0$ has the solution $x = 6$ which leads to $r = 8$.

Puzzle three:

Welcher Wert ist gesucht?

https://www.spektrum.de/raetsel/welcher-wert-ist-gesucht/2156895

Which value is searched for?

"The picture shows two circles with two common tangents, which have lengths 14 and 16. What is the area of a rectangle whose sides are as long as the radii of the two circles?"
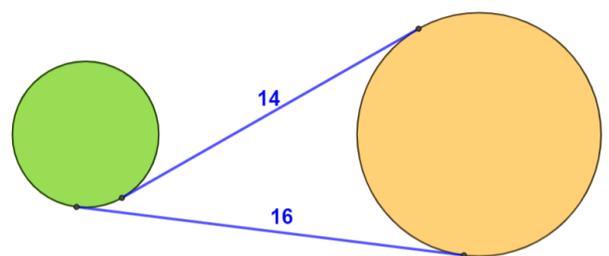
See Wolfgang's answer and his additional tasks for colleagues and students as well:

Solution:

From the adjoining drawing the solution results from the two shown right-angled triangles:

Triangle ABC:

$$\overline{AC}^2 = 16^2 + (R-r)^2$$

Triangle ACD:

$$\overline{AC}^2 = 14^2 + (R+r)^2$$
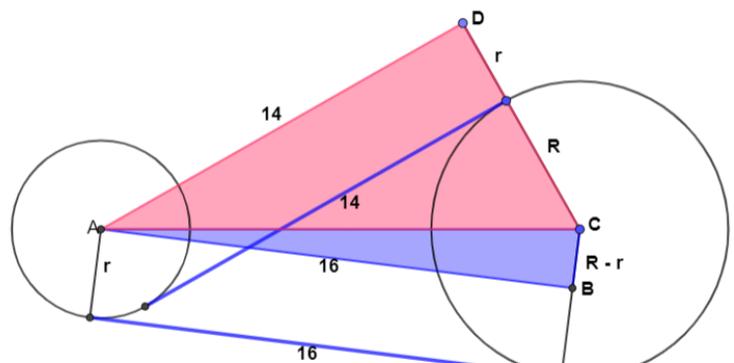
From this follows:

$$256 + R^2 - 2R \cdot r + r^2 = 196 + R^2 + 2R \cdot r + r^2$$
$$60 = 4R \cdot r$$
$$\left. \right\} \rightarrow \boxed{R \cdot r = 15}$$

For $R = 5$ and $r = 3$ one receives with a distance of the centers $\overline{AC} = 2 \cdot \sqrt{65}$ according to the construction the tangents on both circles with the requested lengths 16 and 14.

Additional tasks:

a) Find the points of tangency P_1 through P_4! Take the origin in A.

b) Describe how to construct the inner and outer tangents of two circles which don't intersect.
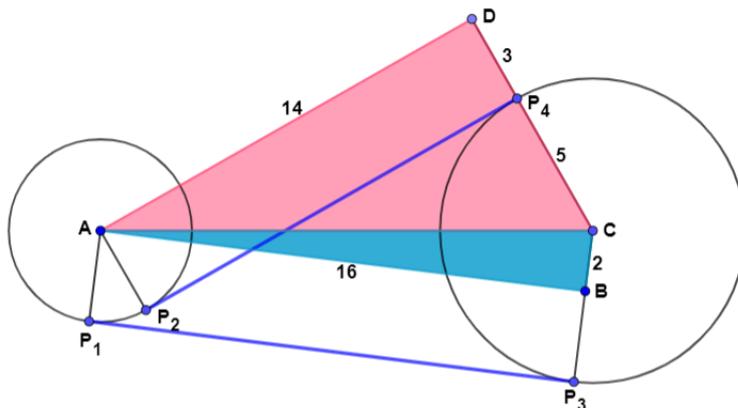
Solution for a)

Equation of the circle with its center in A: $K_1 : x^2 + y^2 = 9$

P_1 results as intersection point of K_1 with straight line $g_1: y = 8x$.

Equation $x^2 + 64x^2 = 9$ leads to

$$\boxed{\begin{aligned} x_1 &= -\frac{3 \cdot \sqrt{65}}{65} \approx -0.3721 \\ y_1 &= -\frac{24 \cdot \sqrt{65}}{65} \approx -2.9768 \end{aligned}}$$

P_2 is the intersection point of K_1 with line $g_2 : y = -\frac{7}{4}x$.

Equation $x^2 + \frac{49}{16}x^2 = 9$ gives

$$\boxed{\begin{aligned} x_2 &= \frac{12 \cdot \sqrt{65}}{65} \approx 1.4884 \\ y_2 &= -\frac{21 \cdot \sqrt{65}}{65} \approx -2.6047 \end{aligned}}$$

Equation of the circle with its center in B: $K_2 : \left(x - 2\sqrt{65}\right)^2 + y^2 = 25$.

P_3 is the intersection point of K_2 with line $g_3 : y = 8\left(x - 2\sqrt{65}\right)$.

The solutions of equation $\left(x-2\cdot\sqrt{65}\right)^2+64\cdot\left(x-2\cdot\sqrt{65}\right)^2=25$ are:

$$x_3=2\cdot\sqrt{65}-\sqrt{\frac{5}{13}}\approx15.50434$$

$$y_3=-8\cdot\sqrt{\frac{5}{13}}\approx-4.9614$$

P_4 is the intersection point of K_2 with line $g_3:y=-\frac{7}{4}\left(x-2\cdot\sqrt{65}\right)$

Solutions of equation $\left(x-2\cdot\sqrt{65}\right)^2+\frac{49}{16}\cdot\left(x-2\cdot\sqrt{65}\right)^2=25$ are:

$$x_4=\frac{22}{13}\cdot\sqrt{65}\approx13.6438$$

$$y_4=\frac{7}{13}\cdot\sqrt{65}\approx4.34122$$

Solution for b)

In my school days, this was a standard task!

The outer tangents:

➢ Two auxiliary circles are needed:
  a) One with center M in midpoint of $\overrightarrow{AB}$ with radius $r=\overrightarrow{MB}$ and
  b) One with center B and the difference of the radii of the given circles as radius.
➢ Draw two segments connecting A with the intersection points of the two auxiliary circles
➢ Move the segments outward until they touch the large (and the small) circle.
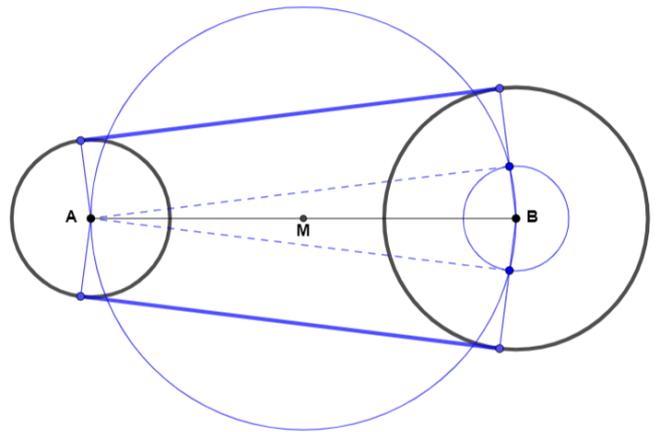
The displaced lines are the common outer the tangents to both circles

The inner tangents:

Construction is similar, so is not described once more.

The difference is, that here the radius of the second auxiliary circle is the sum of the two given circles.
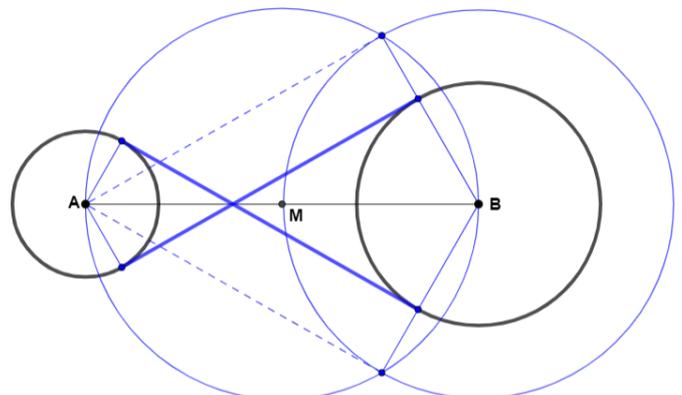
As you can imagine I was inspired by Wolfgang's additional tasks to contribute with additional tasks, too.

The first one is to produce a model with variable radius r for the smaller circle. r can be varied by a slider. I have predefined $ac := \sqrt{16 + \left(\dfrac{15}{r} - r\right)^2}$
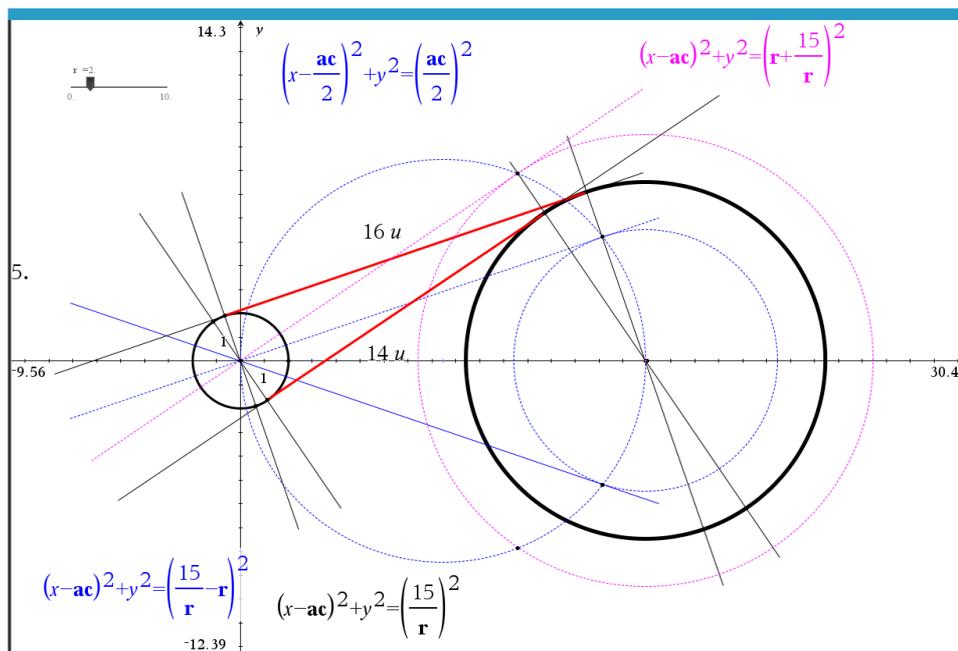
My second task can train the student's competency working with a CAS:

Generalize the problem with a and b as the lengths of the segments between the circles and r and R as the radii of the circles.

I start following Wolfgang's calculation. Fortunately, I have his solution as a very welcome reference to check my procedure.

```
#1:    [CaseMode := Sensitive, InputMode := Word]

#2:    [R :∈ Real (0, ∞), a :∈ Real (0, ∞), b :∈ Real (0, ∞)]
```

$$\#3:\quad a^2 + (R - r)^2 = b^2 + (R + r)^2$$

$$\#4:\quad R^2 - 2 \cdot R \cdot r + a^2 + r^2 = R^2 + 2 \cdot R \cdot r + b^2 + r^2$$

$$\#5:\quad R^2 - 2 \cdot R \cdot r + a^2 + r^2 - (R^2 + 2 \cdot R \cdot r + b^2 + r^2) = 0$$

$$\#6:\quad -4 \cdot R \cdot r + a^2 - b^2 = 0$$

$$\#7:\quad R \cdot r = \frac{a^2 - b^2}{4}$$

#8: Check

#9: $\text{SUBST}\left(R \cdot r = \dfrac{a^2 - b^2}{4}, [a, b], [16, 14]\right) = (R \cdot r = 15)$

#10: $r := \dfrac{a^2 - b^2}{4 \cdot R}$

#11: $\left(AC := \sqrt{a^2 + (R - r)^2}\right) = AC := \dfrac{\sqrt{(16 \cdot R^4 + 8 \cdot R^2 \cdot (a^2 + b^2) + (a^2 - b^2)^2)}}{4 \cdot R}$

#12: $\text{SUBST}(AC, [R, a, b], [5, 16, 14]) = \boxed{2 \cdot \sqrt{65}}$

I apply the "Hesse form" for a line $y = k\,x + d$. The distance of the lines from the center of the circle is the circle radius r respective R. The root in the denominator can be positive or negative.

$\text{SUBST}\left(r = \dfrac{y1 - k \cdot x1 - d}{\sqrt{(1 + k^2)}}, [x1, y1], [0, 0]\right) = \left(\dfrac{a^2 - b^2}{R} = -\dfrac{4 \cdot d}{\sqrt{(k^2 + 1)}}\right)$

$\text{SUBST}\left(R = \dfrac{y2 - k \cdot x2 - d}{\sqrt{(1 + k^2)}}, [x2, y2], [AC, 0]\right) = \left(R = -\dfrac{k \cdot \sqrt{(16 \cdot R^4 + 8 \cdot R^2 \cdot (a^2 + b^2) + (a^2 - b^2)^2)} + 4 \cdot R \cdot d}{4 \cdot R \cdot \sqrt{(k^2 + 1)}}\right)$

$\text{SOLVE}\left(\left[\dfrac{a^2 - b^2}{R} = -\dfrac{4 \cdot d}{\sqrt{(k^2 + 1)}}, R = -\dfrac{k \cdot \sqrt{(16 \cdot R^4 + 8 \cdot R^2 \cdot (a^2 + b^2) + (a^2 - b^2)^2)} + 4 \cdot R \cdot d}{4 \cdot R \cdot \sqrt{(k^2 + 1)}}\right], [k, d]\right)$

$\left[k = -\dfrac{4 \cdot R^2 - a^2 + b^2}{4 \cdot R \cdot a} \wedge d = \dfrac{(b^2 - a^2) \cdot \sqrt{(16 \cdot R^4 + 8 \cdot R^2 \cdot (a^2 + b^2) + (a^2 - b^2)^2)}}{16 \cdot R^2 \cdot a} \wedge R \cdot \sqrt{(k^2 + 1)} \neq 0\right]$

Taking k and d for slope and intercept gives the first tangent and its intersection with the two circles the osculation points P_1 and P_3.

$\text{circ1} := x^2 + y^2 = r^2$

$\text{tang1} := y = -\dfrac{4 \cdot R^2 - a^2 + b^2}{4 \cdot R \cdot a} \cdot x + \dfrac{(b^2 - a^2) \cdot \sqrt{(16 \cdot R^4 + 8 \cdot R^2 \cdot (a^2 + b^2) + (a^2 - b^2)^2)}}{16 \cdot R^2 \cdot a}$

$(\text{SOLUTIONS}([\text{circ1}, \text{tang1}], [x, y]))_1$

$\left[\dfrac{(b^2 - a^2) \cdot (4 \cdot R^2 - a^2 + b^2)}{4 \cdot R \cdot \sqrt{(16 \cdot R^4 + 8 \cdot R^2 \cdot (a^2 + b^2) + (a^2 - b^2)^2)}}, \dfrac{a \cdot (b^2 - a^2)}{\sqrt{(16 \cdot R^4 + 8 \cdot R^2 \cdot (a^2 + b^2) + (a^2 - b^2)^2)}}\right]$

$[-0.3721042037, \ -2.976833630]$

```
            2    2     2
circ2 := (x - AC) + y  = R
```

```
(SOLUTIONS([circ2, tang1], [x, y]))
                                    1
```

$$\left[\frac{4 \cdot R^2 \cdot (3 \cdot a^2 + b^2) + a^4 - 2 \cdot a^2 \cdot b^2 + b^4}{4 \cdot R \cdot \sqrt{(16 \cdot R^4 + 8 \cdot R^2 \cdot (a^2 + b^2) + (a^2 - b^2)^2)}}, \ - \frac{4 \cdot R^2 \cdot a}{\sqrt{(16 \cdot R^4 + 8 \cdot R^2 \cdot (a^2 + b^2) + (a^2 - b^2)^2)}}\right]$$

$[15.50434182, -4.961389383]$

Changing one sign and performing the same procedure as above leads to points P_2 and P_4 as expected:

$$\text{SOLVE}\left(\left[\frac{a^2 - b^2}{R} = -\frac{4 \cdot d}{\sqrt{(k^2 + 1)}}, \ R = \frac{k \cdot \sqrt{(16 \cdot R^4 + 8 \cdot R^2 \cdot (a^2 + b^2) + (a^2 - b^2)^2)} + 4 \cdot R \cdot d}{4 \cdot R \cdot \sqrt{(k^2 + 1)}}\right], \ [k, d]\right)$$

…

…

The generalized results enable us to construct own problems with other values given, e.g.:

"The picture shows two circles with two common tangents, which have lengths 12 and 9.

What is the area of a rectangle whose sides are as long as the radii of the two circles?"
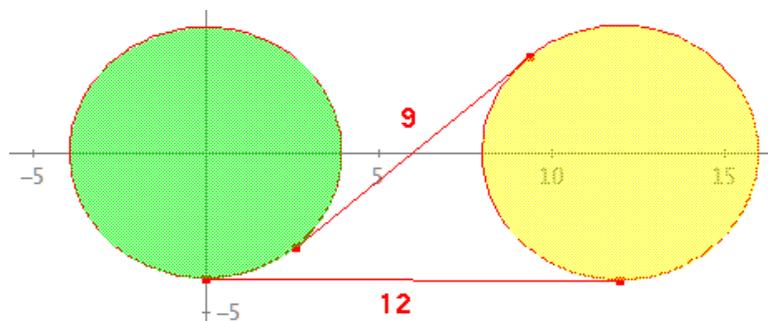
(Solution: $A = 15.75$)
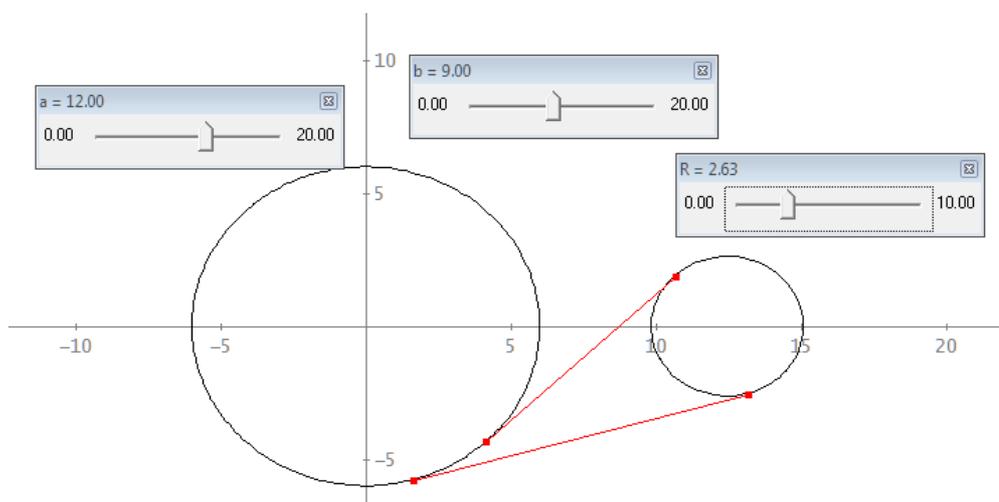
From Mandelbrot to Mandelpinskis

Josef Böhm, Würmla, AUT

In "Spektrum der Wissenschaft 7.23" was an article written by Christoph Pöppe about „Mandel-pinskis". The expression „Mandelpinski" was created by Robert L. Devaney for an interesting family of dynamic systems with iteration functions of the form $f(z) = z^n + \dfrac{c}{z^m}$ or $f(z) = z^n + \dfrac{d^{n+m}}{z^m}$. (See the highly recommended references at the end of this article.)

I was fascinated by the pictures which illustrated the article and wanted to reproduce the pictures of the Mandelbrot sets – and to experiment with own parameters - together with the respective Julia sets.
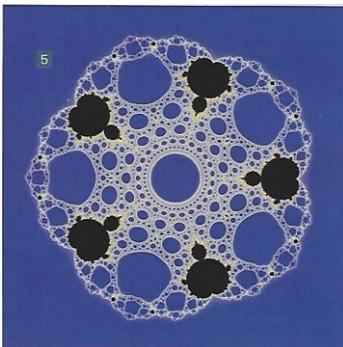
This is the Mandelbrot set for $f(z) = z^2 + \dfrac{d^6}{z^3}$.

(Figure from "Spektrum)

I wanted to reproduce one or the other of these wonderful pictures. There are a lot of tools to plot Julia Sets (see Roman Hasek's contribution in this DNL and many others), but I didn't find programs to plot Mandelbrot Sets based on own iteration functions.

Pöppe presents three pictures of Julia Sets for $f(z) = z^4 + c/z^4$ with $c = -0.1$, $c = 0.04$ and $c = 0.2$ (from left to right):
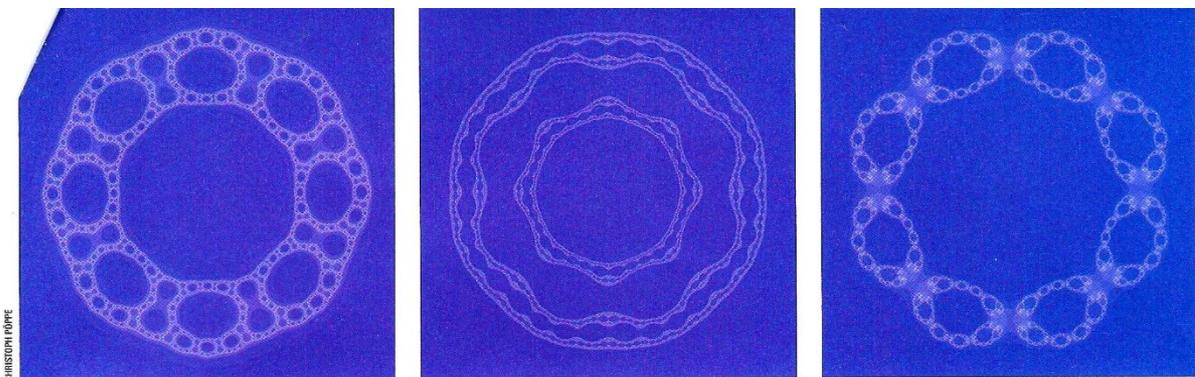
I tried Roman's function, but I failed calculating the Julia Sets. The functions seem to be too complex (in both senses of the word).

Finally, I came across a program which looked very promising:

fractal zoomer.tns for TI-NspireCAS written by Jens Kolbinger, which can be downloaded for free from

https://www.ticalc.org/archives/files/fileinfo/463/46381.html

You need the side screen, because the Menu button activates the top-down menus for Settings, View (Info, Zoom In, Zoom Out, …) and Colouring (offering various colour palettes). Handheld Document Preview is recommended.

I started reproducing the Julia Sets presented in the journal and followed the advice given in the readme.txt file defining the iteration formula in the Calculator App as f:=.

The Julia Sets are displayed in an instant.

I was satisfied when I compared my results with the pictures given in the "Spektrum". In the following I'd like to explain how to work with this program

In the first line of Option Settings you are offered three options: mandelbrot, julia and custom. The options can be accessed by the arrow buttons. Option custom allows plotting self-defined Julia Sets – **but not** Mandelbrot sets!

When you start with the settings given you will meet the famous "Little Apple Man":
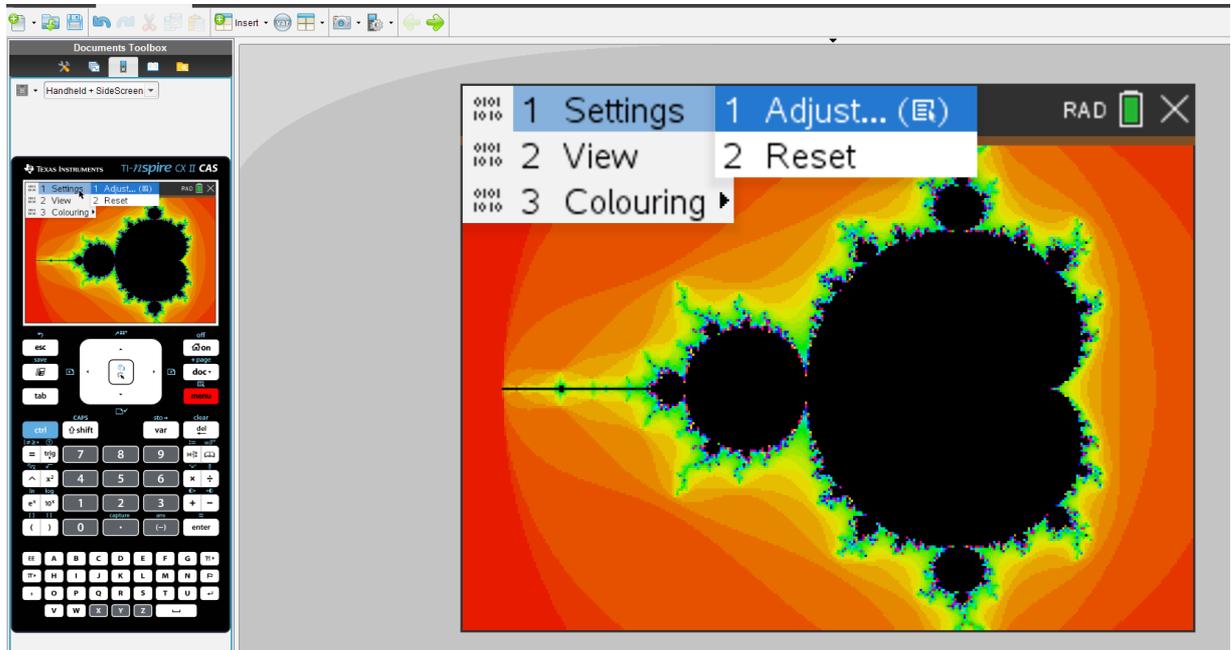
You need the side screen, because the Menu button activates the top-down menus.
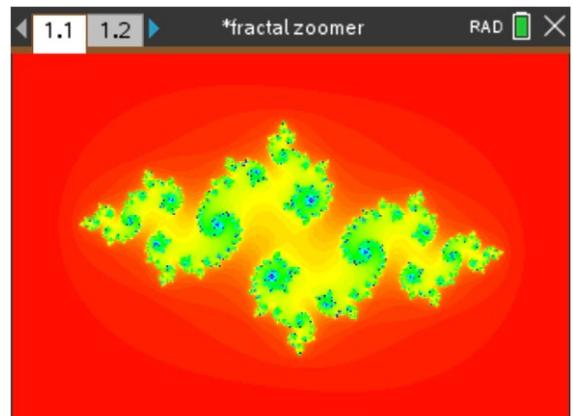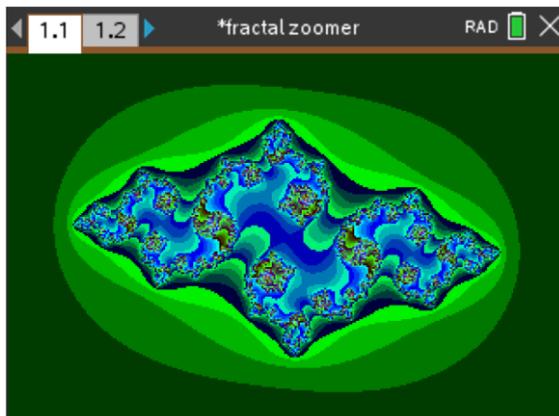
This was pretty nice – but I was really not satisfied, because I wanted to plot my own Mandelbrot Sets. fractal zoomer.tns allows only to plot the classical Mandelbrot Set with its respective Julia Sets with different values for c (here for $c = -0.8 + 0.2i$).

Now comes the hard part. The program author of fractal zoomer.tns invites in his readme.txt to contact him for questions. I tried several times, but didn't receive any answer. The program is a LUA script and I started to investigate the code. The script comprises 800 lines of code 😊.

Studying carefully line for line I found the magic line. Line 499 sets the instruction for the iteration $f(z) = z^2 + c$. But we have to define the function with real part and imaginary part separated as Zr and Zi in the LUA-syntax defined as pair $Zr,Zi = \dots$

```
490     if fractal==1 then      --mandelbrot set
491         for Py=0,WH,res do
492             Pi=getImaginaryOfY(Py)
493             if results[Px][Py] then
494                 column[Py]=results[Px][Py]
495             else
496                 column[Py]=0
497                 Zr,Zi=Z0r,Z0i
498                 for i=1,maxI do
499                     Zr,Zi=Zr^2-Zi^2+Pr,2*Zr*Zi+Pi
500                     sqZ=Zr^2+Zi^2
501                     if sqZ>radius then
502                         if smooth then
503                             column[Py]= i + 1 - log(log(sqrt(sqZ)))/log2
504                         else
505                             column[Py]= i
506                         end
507                         break
508                     end
509                 end
510                 results[Px][Py]=column[Py]
511             end
512         end
513     elseif fractal==2 then    --julia set
```

The next large if-block starting with line 524 works similar for Julia Sets and we have to edit this line. On page 8 we plotted the Julia Set for $f(z) = 3z – 3z^2$. How does the respective Mandelbrot Set look like? I prepare the iteration formula for the LUA script.
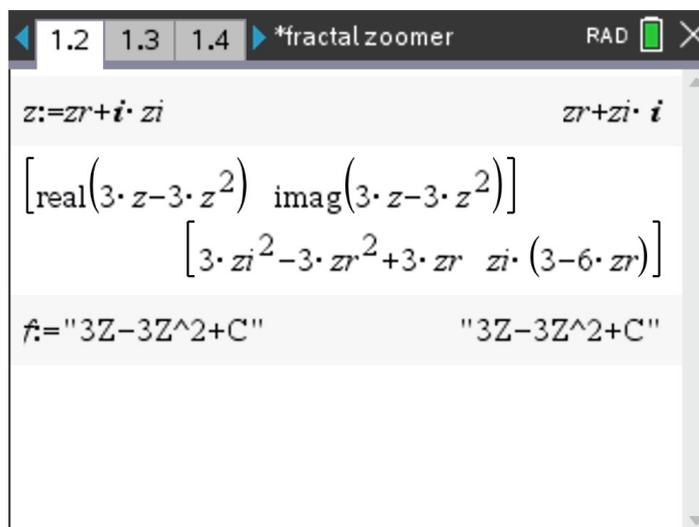
The "- -" at the begin of a code line makes it to a comment – I added a comment in line 499 and re-place line 499 from above by inserting line 502 with the definition obtained above. So, we will not lose the original code. Line 501 is also a comment.

```
499                         -- Original
500                         --Zr,Zi=Zr^2-Zi^2+Pr,2*Zr*Zi+Pi
501                         -- f(z) = 3z - 3z^2
502                         Zr,Zi=3*Zi^2-3*Zr^2+3*Zr+Pr,Zi*(3-6*Zr)+Pi
```

The resulting Mandelbrot Set:
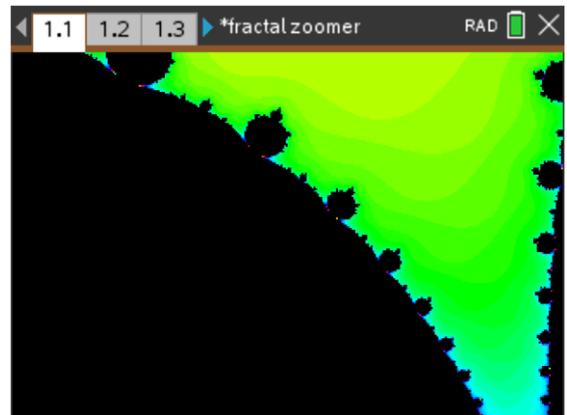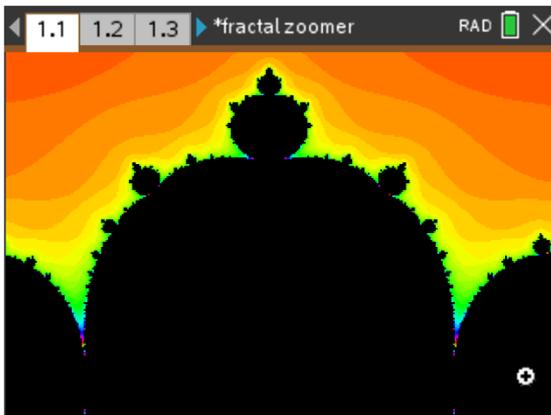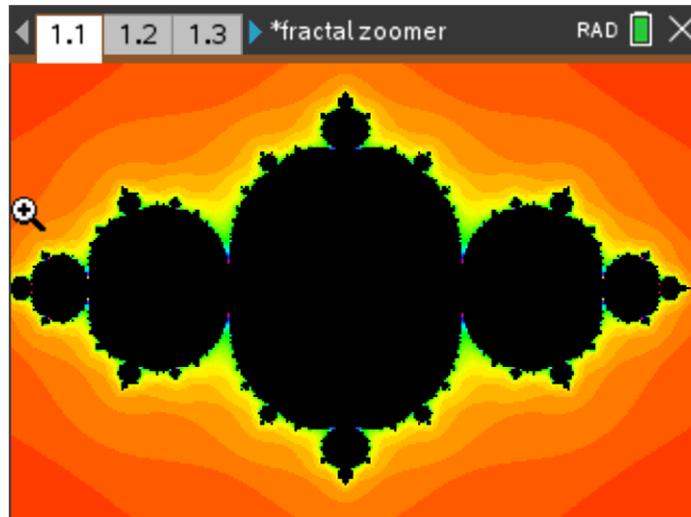
Zooming in might remind us on the "Journey into the Valley of Sea Horses" in a famous video from *Peitgen*, *Jürgens* and *Saupe* produced many years ago.

The right picture is a visit in the original valley in the original Mandelbrot Set.
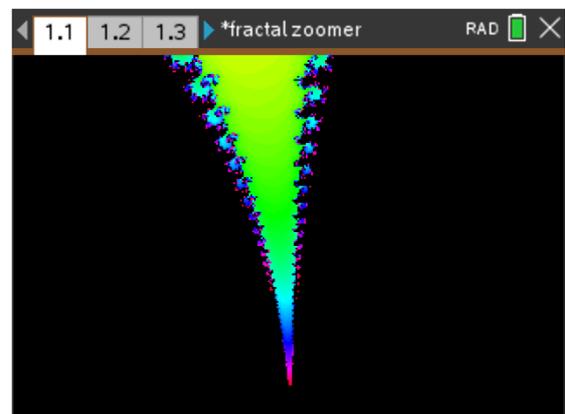

Pöppe mentions a special Julia Set – the Steinmetz-Set, which is important in the theory of modul groups. It has a more complicated iteration function. So, I was excited if fractal zoomer.tns could master this "Challenge": $f(z) = \dfrac{4}{27} \dfrac{(1-z+z^2)^3}{z^2(1-z)^2}$

"Steinmetz Set"   "Steinmetz Set"

$f := $ "4· (1−Z+Z^2)^3/(27· Z^2· (1−z)^2)+C"

         "4· (1−Z+Z^2)^3/(27· Z^2· (1−z)^2)+C"

Two pictures with radius = 1000 and $c = 0 + 0 \cdot i$ in different colourings:
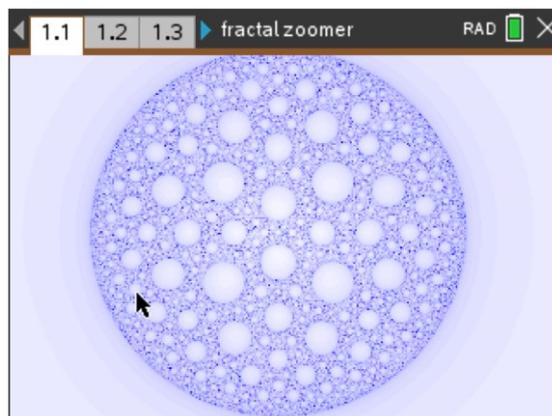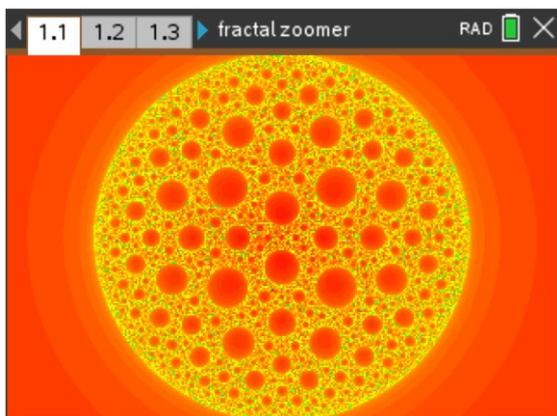
Slight changes in the variable c result in new – unexpected – pictures. Inporting them to a Graphics program like Paint Shop or Photoshop one can create nice posters or similar.
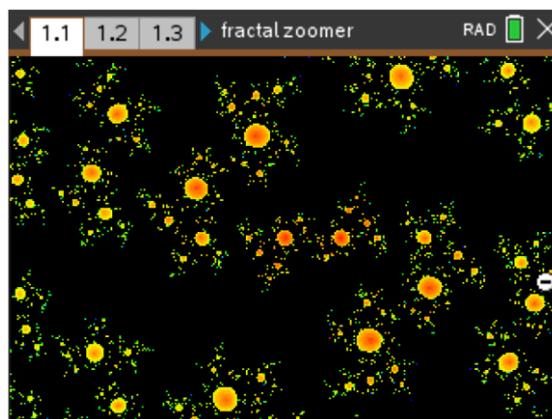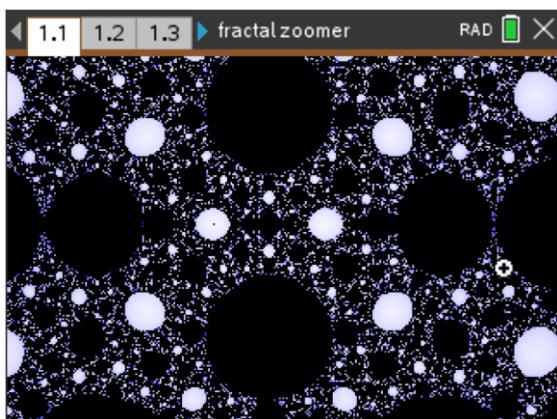
I will turn back to the inspiration of my article, the "*Mandelpinskis*" and start with $f(z) = z^2 + \dfrac{c}{z^2}$.
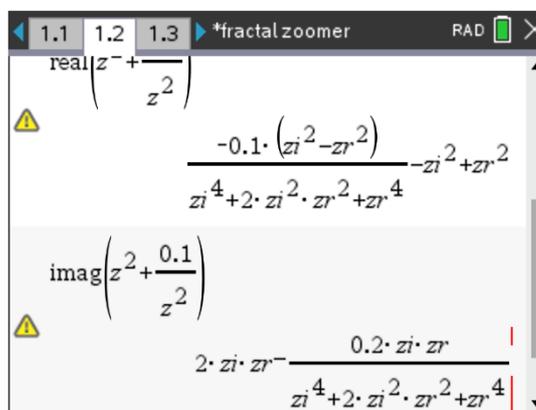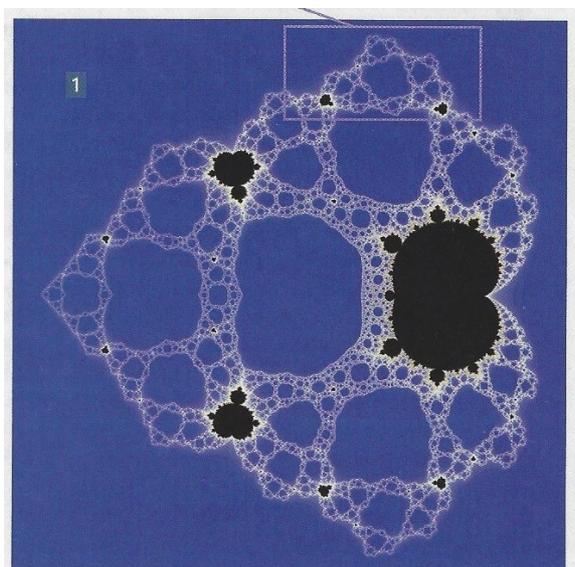

(Figure from "Spektrum")

Then I transfer the expressions for *Zr* and *Zi* into the LUA script – and hope that after pressing the ENTER-button a picture similar to that shown in the journal would appear! (Left figure)

I plotted the Mandelpinski with different values for Z_0:
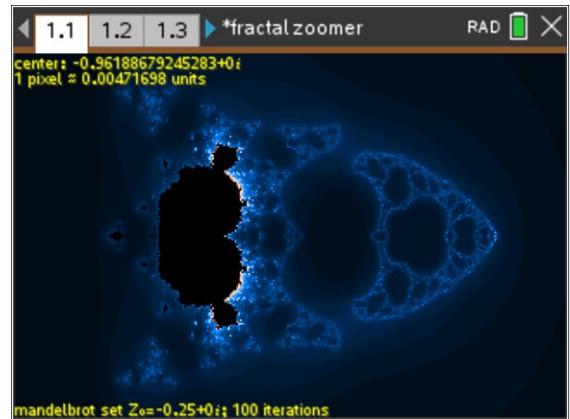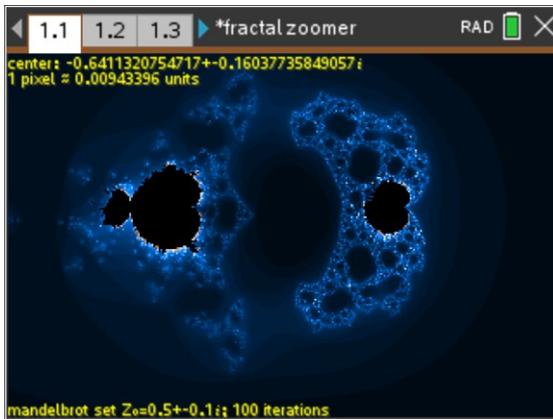

Same figure with changing colours:
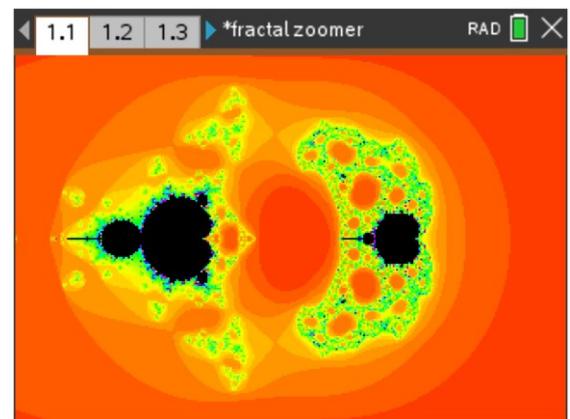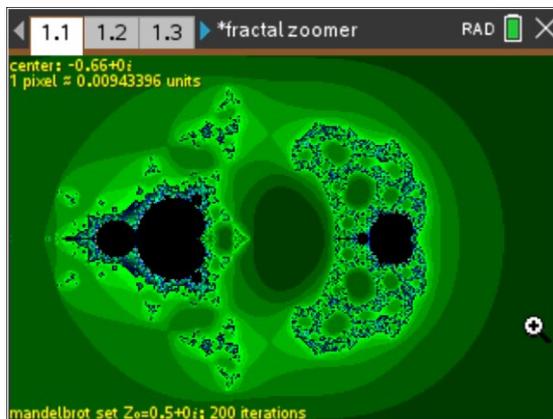
Another initial value Z_0 one together with a Julia Set:
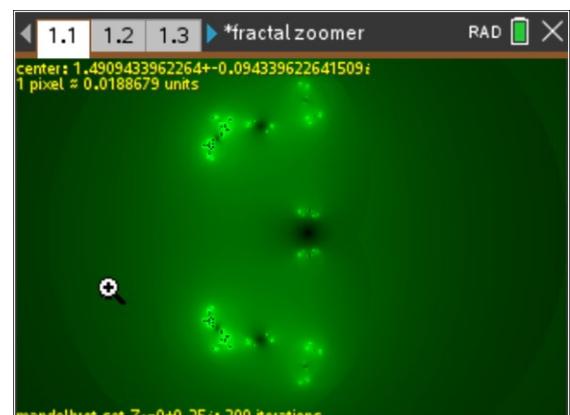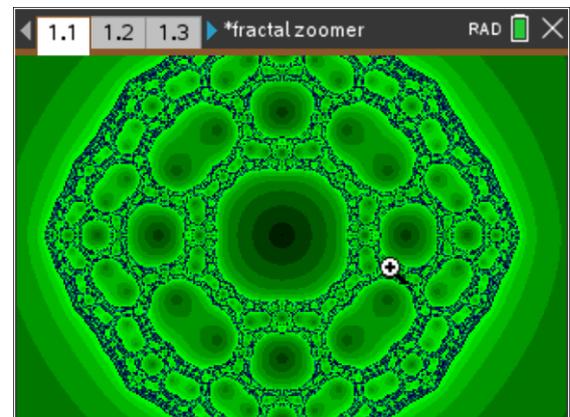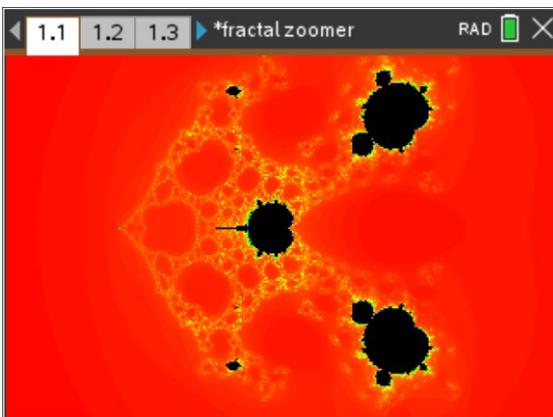
I was satisfied with my pictures. Finally, I will close with some screens from my gallery:
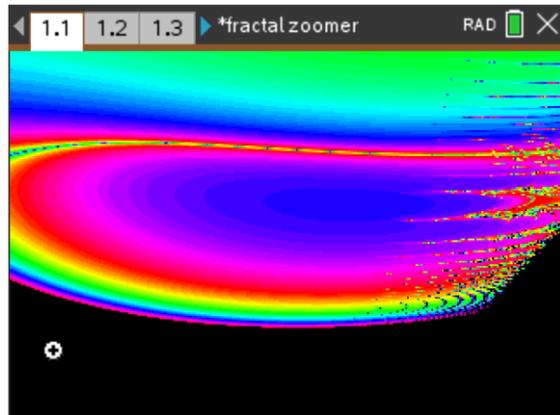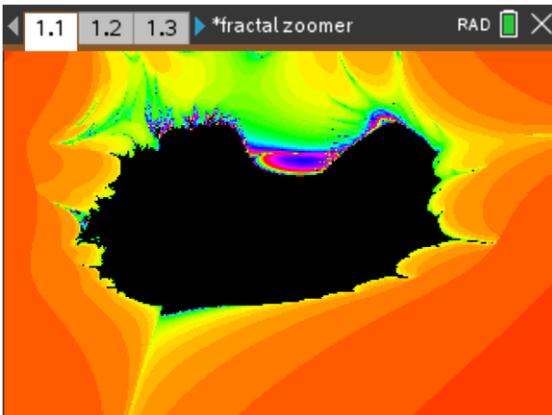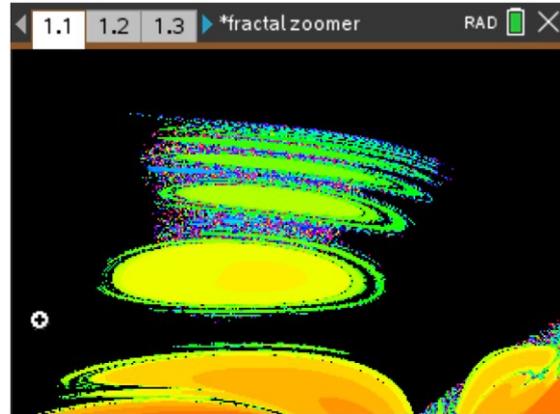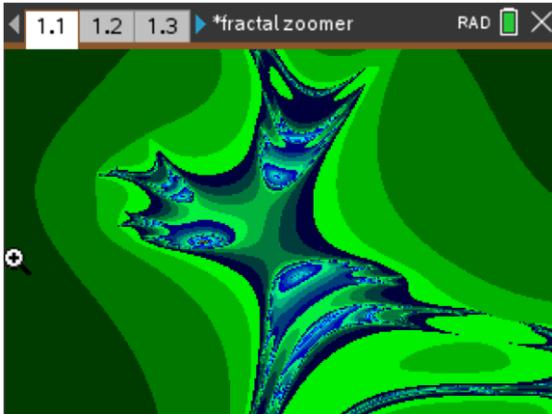
It would be great if you also would experiment with this program and eventually send interesting plots, Josef

Here can you download fractal zoomer.tns. There are also some interesting fractal pictures.
Jens Koblinger, the author of the program writes at the beginning of the LUA script the comment:

-- written by Jens Kolbinger; program and code for free use;

contact: jens.software@gmail.com

https://www.ticalc.org/archives/files/fileinfo/463/46381.html

fractal zoomer_J.tns is the original file + some changes included as comments, Josef

A rich resource of Mandelpinski papers written by Robert Devaney:

http://math.bu.edu/people/bob/papers.html

http://math.bu.edu/people/bob/papers/struc.pdf

http://math.bu.edu/people/bob/slides.html

http://math.bu.edu/people/bob/Mandelpinski-Barcelona.ppt