Circle Angles

Teacher Notes

Introduction

This activity includes seven tasks that ask students to work independently to investigate the properties of angles in circles.

The seven investigations are:

- Triangle on a Chord and Centre
- Perpendicular Bisector of Chords
- Inscribed Angle Theorem
- Angle in a Semicircle
- Tangent to a Circle
- The Tangent Kite
- The Cyclic Quadrilateral

Resources

Students start with a new, blank TI-Nspire document and each investigation may be carried out on a separate page. A **worksheet** lays out the investigations and guides students, step by step, through the necessary constructions.

Skills required

It is assumed that students will be able to carry out the following TI-Nspire processes.

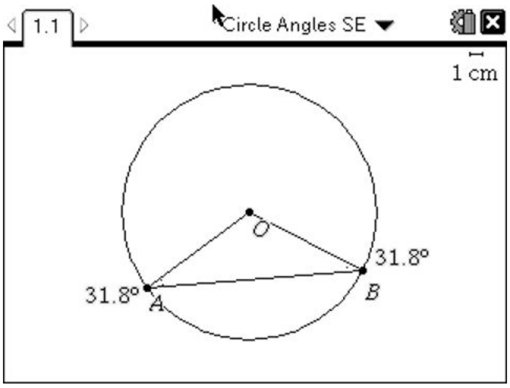
- ✓ Open and save a new tns document.
- ✓ Insert a new page in a document
- ✓ Use the Geometry menus to select commands.
- ✓ Use these options from Shapes menu:
  - 1: Circles
  - 2: Triangles
  - 4: Polygons
- ✓ Use these options from the Points & Lines menu:
  - 1: Point
  - 2: Point On
  - 3: Intersection Point(s)
  - 4: Line
  - 5: Segment
  - 7: Tangent
- ✓ Use these options from the Construction menu:
  - 3: Perpendicular Bisector
  - 5: Midpoint
- ✓ Use these options from the Measurement menu:
  - 1: Length
  - 4: Angle
- ✓ Label points
- ✓ Grab and move points and objects.
- ✓ Hide points

Note: these techniques are *not* described on the worksheet.

The investigations

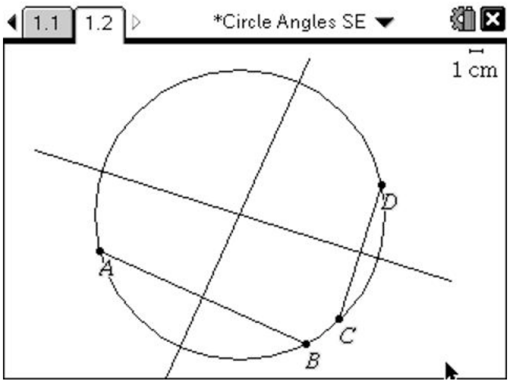
In each case students are able to move points and see how the measurements of angles change. This will convince them that a particular result *always seems to be true*. They are then encouraged to *prove* the result, either using an algebraical or a geometrical approach.

Triangle on a Chord and Centre



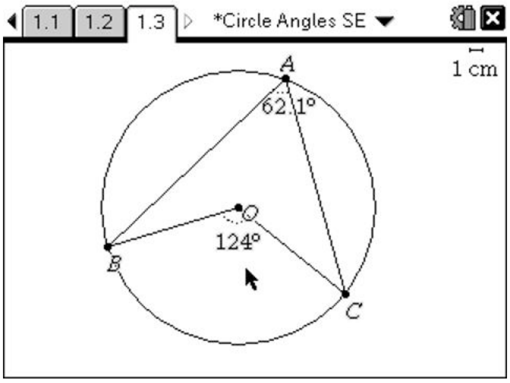
Students are guided, step by step, to create a Geometry page similar to the one shown above. They should realise that as A or B are moved, angles OAB and OBA are always equal. The explanation is that since OA and OB are radii of the circle triangle OAB is isosceles.

Perpendicular Bisector of Chords



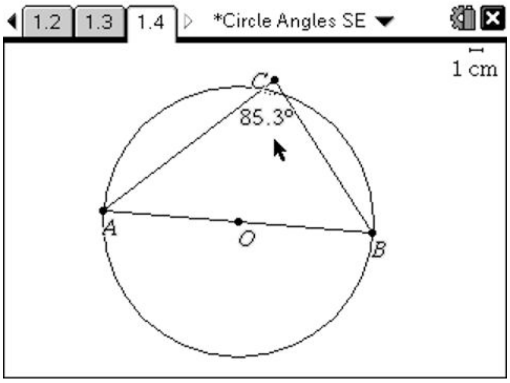
Students produce a Geometry page like this with the centre of the circle hidden. They should notice that as points A, B, C or D are moved the two perpendicular bisectors always intersect at the same point and that this point is the centre of the circle. Students may find it convincing finally to reveal the centre again.

Inscribed Angle Theorem



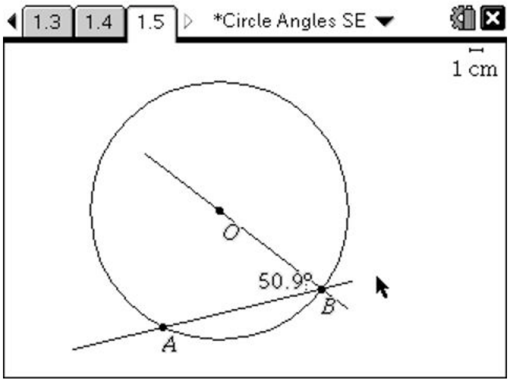
This construction should convince students that the angle at the circumference (the inscribed angle) is always half the angle at the centre. A hint is provided to get them started on an algebraic proof.

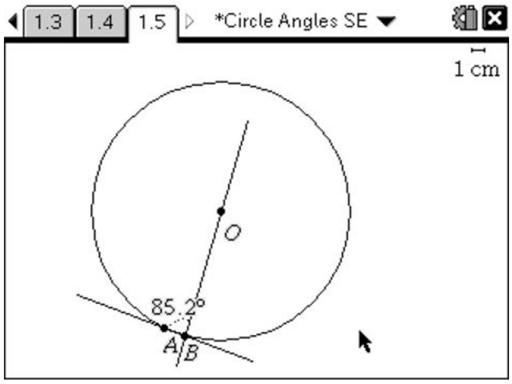
Angle in a Semicircle



This construction starts, not with a circle, but with the segment AB. As students move point C in towards the circle they discover that angle ACB gets closer to 90°. This result links nicely with the previous construction since in this case the angle at the centre is 180°.

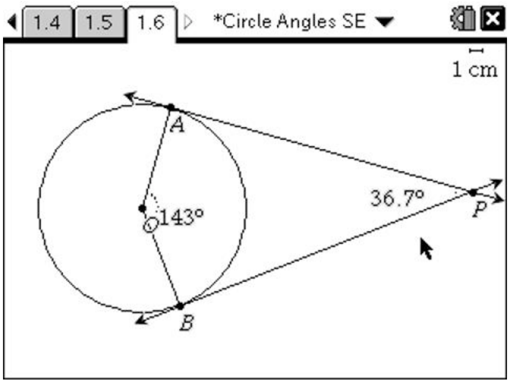
Tangent to a Circle





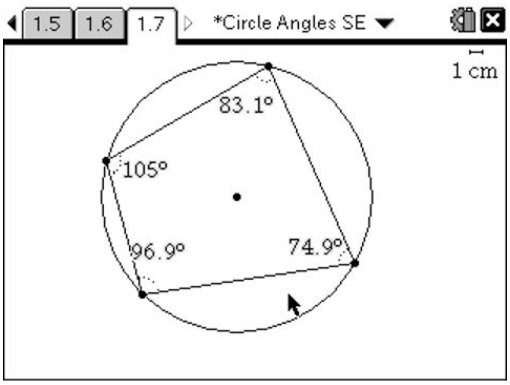
As students move point B around the circle closer to A they should see that the angle between the two lines gets closer to 90°. One explanation for this property relies on the notion of a limit. Consider the angles of the triangle AOB. As B approaches A, the angle at the centre clearly approaches zero so the other two angles must approach 90°. In the limit when B is at A, although the triangle no longer really exists, the angle between the two lines equals 90°.

The Tangent Kite



Students should realise that the sum of angles AOB and APB is 180°
This proof uses the result of the previous investigation that the angle between a tangent and a radius is 90°.

The Cyclic Quadrilateral



Students should realise that the sum of opposite pairs of angles is 180°. An algebraic proof can be constructed using the result of the Inscribed Angle Theorem