Power and Exponential Laws. **Teacher Notes**

Introduction

This activity involves the use of a straight-line graph to confirm relationships of the form $y = ax^{b}$ and y=a.b^x. Students must be able to model mathematically situations involving power or exponential functions eq. from experimental data they may be required to draw a graph of log y against log x and to deduce values of a and b such that $y = ax^{b}$.

TI-Nspire is first used to draw a quick graph of experimental data to establish that there is some kind of connection between the variables. Then the Regression tools in the Analyze menu are used to determine what the relationship is.

TI-Nspire is then used to graph $\log_{10} y$ against $\log_{10} x$ and show that there is a linear connection. This leads to how the equation can be found using mathematical methods without using the advanced TI-Nspire analysis facilities.

There are two parts to the activity:-



log₁₀ y against log₁₀ x (Power Law)



log₁₀ y against x (Exponential Law) •



Resources

The TI-Nspire document PowerLaw deals with relationships of the form $y = ax^{b}$ and graphs $log_{10} y$ against log₁₀ x.

The TI-Nspire document ExponentialLaw deals with relationships of the form $y=a.b^{x}$ and graphs $\log_{10} y$ against x.

There are two PowerPoint presentations to support students/teachers who are unfamiliar with the TI-Nspire, one for each activity.

There is a worksheet for both activities as well as full solutions below.

Skills required

It is assumed that students will be able to carry out the following basic TI-Nspire processes.

- ✓ Open and save a new ths document.
- ✓ Move from one page to another.
- ✓ Use menus to select commands.
- ✓ In a Data & Statistics page change variables on the axes

Other more unusual techniques are described in full in the teacher's notes.

The Power Law activity, plotting $log_{10} y$ against $log_{10} x$.

The TI-Nspire document PowerLawv1.tns is divided into 10 examples.

Example 1. A worked example

Page 1.1 shows a table of values for t and v.

To use quick graph to show a connection between the variables press: (menu) (3) Data (6) Quick Graph



Move the cursor to the middle
of the y-axis.
Press (enter)

∢ 1.1 A t	1.2	2.1 ▶ *F ■ v	PowerLawv1MT 👻 🗐 🗙
•	10		
2	20	17.9	Click or Enter to add variable
3	30	33.0	
4	40	50.6	
5	50	7	
AI	=10	<	> t

∢	1.1 1.2	2.1 🕨 *Pov	verLawv1MT 🔻 🛛 🔞 🔀
	A t	B _V	- 0
٠			60-
1	10	6.3	
2	20	17.9	> 40- . O
3	30	33.0	20-
4	40	50.6	
5	50	7	
4	41 =10	< >	10 20 30 40 30 t

御区

y = 0.20 x^{1.50} 10 15 20 25 30 35 40 45 50

60

40. 20.

Move to page 1.2 (m)

Choose v (enter)

Find the equation connecting the variables: ((menu) (4) Analyse (6) Regression (7) Show Power

Move to page 2.1 where a table of values for log_{10} t and log_{10} v has been constructed.

<u>ات</u> ک 1.1 1.2 2.1 PowerLawv1MT 🔻 logv logt =log('t) =log('v) 10 6.30 0.80 20 17.90 1.30 1.25 30 33 1.48 1.52 50.60 1.60 1.70 40 50 71 1.70 1.85 < > AI 10

Move to page 2.2. Change the variables on the axes to log_{10} t and log_{10} v.

using tab to navigate.

Power and Exponential Laws



Now introduce the mathematical strategies required to find the equations. Students complete the Power Law Worksheet Example 1 along with the teacher. The completed solution for Example 1 is shown below.

POWER LAW WORKSHEET

Example 1

t	10	20	30	40	50
V	6.3	17.9	33.0	50.6	71.0
log ₁₀ t	1.00	1.30	1.48	1.60	1.70
$\log_{10} V$	0.80	1.25	1.52	1.70	1.85

log ₁₀ V	Equation of Straight Line	Equation of Power Function
(1, 0.8)	Y = mX + c	$\log_{10} V = 1.5 \log_{10} t - 0.7$
	$\log_{10} v = 1.5 \log_{10} t + c$	$\log_{10}(?) = -0.\check{7}$
log ₁₀ t	Find y intercept	(?) = 10 ^{-0.7}
Find gradient.	(1, 0.8) lies on the line.	$\log_{10} V = \log_{10} t^{1.5} + \log_{10} (10^{-0.7})$
$m = \frac{y_2 - y_1}{x_1 - x_2}$	0.8 = 1.5 x 1 +c	$\log (1-\log t^{1.5} + \log (0.2))$
$x_2 - x_1$	0.8 - 1.5 = c	$\log_{10} v = \log_{10} t + \log_{10} (0.2)$
$=\frac{1.7-0.8}{1.6-1}$	c = -0.7	$\log_{10} V = \log_{10} \left(t^{1.5} \ge 0.2 \right)$
$=\frac{0.9}{0.6}=1.5$	log ₁₀ V = 1.5 log ₁₀ t - 0.7	$V = 0.2 t^{1.5}$

Example 2

This is another worked example. This time the equation of the straight line is worked out first using mathematics and it is then checked using the Regression tool of the handheld.

Move to page 3.1 where a table of values for $x,h,log_{10} x$ and $log_{10} h$ has been constructed.

4 2.1 2.2 3.1 ▶ *PowerLawv1MT ▼ 100 Km × 100 Km									
A x	В	h	⊂ _{logx}	logh					
1	1	2	-10g(X) 0	0.30	-				
2	2	16	0.30	1.20	-				
3	3	54	0.48	1.73					
4	4	128	0.60	2.11					
5	5	250	0.70	2.40	V				
AI 1				<	>				

To check the equation of the straight line move to page 3.2

(menu) (4) Analyse
(6) Regression
(1) Show Linear(mx+b)







Power and Exponential Laws



Examples 3 and 4

Students then work on Examples 3 and 4, finding the equations first using mathematics then using the handheld to check the answers.





				-
•			=log('g)	=log('d)
1	1	20	0	1.30
2	3	2.22	0.48	0.35
3	5	0.80	0.70	-0.10
4	7	0.41	0.85	-0.39
5	9	0.25	0.95	-0.60
AI	1			<

Examples 5 to 10.

-2.00·x+1.30 0.6 pgo 0.0 -0.6 0.4 0.6 logg 0.0 0.2 0.8



0

6.

Page 1 of each example shows the table of values.

Students move to page 2 to see that there is a connection between the variables and find the equations using mathematics.



Power and Exponential Laws

These equations are then checked on pages 3 and 4.



The Exponential Law activity, plotting $log_{10} y$ against x.

The TI-Nspire document ExponentialLawv1.tns is divided into 7 Examples.

Example 1. A worked example

Page 1.1 shows a table of values for t and v.

To use quick graph to show a connection between the variables press: (menu) (3) Data (6) Quick Graph

Move the cursor to the middle

of the y-axis. Press (enter)

Choose v (enter)







Move to page 1.2 (tr)

Find the equation connecting the variables
(menu) (4) Analyse
(6) Regression
(8) Show Exponential



Mathematics can now be used to find the equation.

Students complete the Exponential Law Worksheet Example 1 along with the teacher. The completed solution for Example 1 is shown on the next page.

EXPONENTIAL LAW WORKSHEET

Example 1

TI-*nspire*

t	1	1.5	2.2	2.5	3
V	6	8.5	13.8	16.9	24
t	1	1.5	2.2	2.5	3
$\log_{10} V$	0.78	0.93	1.14	1.23	1.38

log ₁₀ V	Equation of Straight Line	Equation of Exponential Function			
(3, 1.38)	Y = mX + c	log ₁₀ V = 0.3 t + 0.48			
	$\log_{10} V = 0.3 t + c$	$\log_{10}(?) = 0.3$ $\log_{10}(?) = 0.48$			
L'	Find y intercept	$(?) = 10^{-0.0}$ $(?) = 10^{-0.00}$			
	(1 , 0.78) lies on the line.	$\log_{10} V = \log_{10} (10^{0.5}) t + \log_{10} (10^{0.46})$			
$m = \frac{y_2 - y_1}{x_2 - x_1}$	$0.78 = 0.3 \times 1 + c$	$\log_{10} V = \log_{10} (2.0) t + \log_{10} (3.0)$			
$=\frac{1.38-0.78}{1.38-0.78}$	c = 0.48	$\log_{10} V = t \log_{10} (2.0) + \log_{10} (3.0)$			
3-1 = 0.6 = 0.3	log ₁₀ V = 0.3 t + 0.48	$\log_{10} V = \log_{10} (2.0)^{t.} + \log_{10} (3.0)$			
$-\frac{1}{2} - 0.5$		$\log_{10} V = \log_{10} \left((2.0)^t \times 3.0 \right)$			
		$V = 3.0 (2.0)^{t}$			

Note that the gradient here needs to be correct to 2 d.p. as all log values have been rounded to this degree of accuracy. Examples 2 to 7 need to be rounded.

Move to page 2.2 ctrl

Check the equation of the straight line. (menu) (4) Analyse (6) Regression (1) Show Linear(mx+b)

Examples 2 to 7.

Page 1 of each example shows the table of values.

Students move to page 2 to see that there is a connection between the variables and find the equations using mathematics.





Power and Exponential Laws

These equations are then checked on pages 3 and 4.



POWER LAW WORKSHEET SOLUTIONS

Example 1

	t	1	0	20	30)	4	0	50
	V	6.	3	17.9	33	.0	50).6	71.0
		1							
	og ₁₀ t	1.0	00	1.30	1.4	8	1.0	60	1.70
l	og ₁₀ V	3.0	30	1.25	1.5	52	1.	70	1.85
			Equ	ation of Straight	t Line	E	Equatior	n of Pow	ver Function
(1, 0.8)			Y	∕ = mX + c		log ₁₀	V = 1.	5 log ₁₀	t-0.7
I			$\log_{10} v = 1.5 \log_{10} t + c$				log ₁₀ ('	(2) = -0.7	
_				Find y intercep	t			?)	?) = 10 ^{-0.7}
Find gradient.		(1, 0.8) lies on the line.		$\log_{10} V = \log_{10} t^{1.5} + \log_{10} (10^{-0.7})$					
$m = \frac{y_2 - y_1}{y_1}$		0.8 = 1.5 x 1 +c				4 5			
$x_2 - x_1$		0.8 - 1.5 = c		$\log_{10} V = \log_{10} t^{1.5} + \log_{10} (0.2)$			+ log ₁₀ (0.2)		
$=\frac{1.7-0.8}{1.6}$		$\frac{-0.8}{4}$ c = -0.7			$\log_{10} V = \log_{10} (t^{1.5} \times 0.2)$			x 0.2)	
	- 0.9	- 1 5	log ₁₀	$V = 1.5 \log_{10} t$	t - 0.7		V = 0	ר 2 t ^{1.}	5
	$-\frac{100}{0.6}$	- 1.0					v – (J. 2 l	

Example 2

X	1		2	3		4	5
Н	2) -	16	54	1	128	250
log ₁₀ x	C)	0.30	0.4	8	0.60	0.70
log ₁₀ H	0.3	30	1.20	1.7	' 3	2.11	2.40
$\begin{bmatrix} \log_{10} H \\ 0, 0.3 \end{bmatrix}$ $\begin{bmatrix} Find grad \\ m = \frac{y_2 - y_2}{x_2 - y_2} \\ = \frac{2.4}{0.7}$ $= \frac{2.1}{0.7}$	(0.7, 2.4) $10g_{10} \times$ ient. $\frac{V_1}{X_1}$ -0.3 -0 = 3	Equa log ₁₀ (0 , 0.3 c = 0.3 log ₁₀	ation of Straight Y = mX + c $H = 3 \log_{10} x + c$ Find y intercep 3) lies on the lin $H = 3 \log_{10} x + c$	<u>t Line</u> + C <u>t</u> e. + 0.3	E log ₁₀ log ₁₀ log ₁₀	Equation of Pow $H = 3 \log_{10} x^{4}$ $\log_{10} (x^{3} + 1)$ $H = \log_{10} x^{3} + 1$ $H = \log_{10} (x^{3} + 1)$ $H = \log_{10} (x^{3} + 1)$ $H = 2 x^{3}$	wer Function (+ 0.3) (+ 0.3) $(- 10^{0.3})$ $(+ 100_{10} (10^{0.3}))$ $(+ 100_{10} (2))$ $(+ 100_{10} (2))$ (+ 20) (+ 20

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Example 3

V	10	20	30	40	50
Р	9.5	16.5	22.8	28.7	34.3
og ₁₀ v	1.00	1.30	1.48	1.60	1.70
og ₁₀ P	0.98	1.22	1.36	1.46	1.54

log ₁₀ P	Equation of Straight Line	Equation of Power Function
(1.7, 1.54)	Y = mX + c	$\log_{10} P = 0.8 \log_{10} v + 0.18 P$
	$\log_{10} P = 0.8 \log_{10} v + c$	$\log_{10}(?) = 0.18$
	Find y intercept	(?) = 10 ^{0.18}
Find gradient.	(1,0.98) lies on the line.	$\log_{10} P = \log_{10} v^{0.8} + \log_{10} (10^{0.18})$
$m = \frac{y_2 - y_1}{x_2 - x_1}$	0.98 = 0.8 x 1 +c 0.98 -0.8 = c	$\log_{10} P = \log_{10} v^{0.8} + \log_{10} (1.5)$
$=\frac{1.54-0.98}{1.7-1}$	c = 0.18	$\log_{10} P = \log_{10} (v^{0.8} x 1.5)$
$=\frac{0.56}{0.7}=0.8$	$\log_{10} P = 0.8 \log_{10} v + 0.18$	$P = 1.5 v^{0.8}$

Example 4

g	1		3	5		7	9
D	2	0	2.22	0.8	30	0.41	0.25
	1			1			
log ₁₀ g	C)	0.48	0.7	' 0	0.85	0.95
log ₁₀ D	1.3	30	0.35	-0.1	10	-0.39	-0.60
$\begin{array}{c} \log_{10} D \\ (0, 1.3) \end{array} \qquad \qquad \underbrace{\text{Equation of Straight Line}}_{Y = mX + c} \end{array}$				<u>E</u> log ₁₀	Equation of Por $D = -2 \log_{10} g$	wer Function g + 1.3	
(0.95 , -0.6) IOg ₁₀			$g_{10} D = -2 \log_{10} g + c$ Find y intercept			iog ₁₀ (?) = 10 ^{1.3}
Find grad	Find gradient. $(0, 1.3)$ lies on the $V_{0} = V_{0}$ $0 = 1.2$		3) lies on the lin	ie.	log ₁₀	$D = \log_{10} g^{-2}$	+ log ₁₀ (10 ^{1.3})

$$m = \frac{y_2 - y_1}{x_2 - x_1}$$

= $\frac{1.3 - (-0.6)}{0 - 0.95}$
= $\frac{1.9}{-0.95} = -2$
(0, 1.3) here on the line.
$$c = 1.3$$

$$\log_{10} D = -2 \log_{10} g + 1.3$$

$$\log_{10} D = \log_{10} g^{-2} + \log_{10} (20)$$

$$\log_{10} D = \log_{10} (g^{-2} \times 20)$$

$$D = 20 g^{-2}$$

Examples 5 to 10

For each example

(i) show that the formula connecting y and x is of the form $y = kx^n$ (on page 2 of handheld).

(ii) find the value of k and n, and state the formula that connects x and y.

Check the equation of the straight line (page 3) and the power function (page 4) on the handheld.

5).	Х		1.26	1.58		2.00	2.50	3.16	
- /	у		3.98	7.94		17.78	31.60	63.10	
	0g ₁₀ x 0g ₁₀ y		0.10	Assume first and last point lie on line of b				0.50	
$\begin{bmatrix} \log_{10} y \\ 0.5, 1.8 \\ 0.5, 1.$			Equation Y = $\log_{10} y =$ Fin (0.1, 0.6) $0.6 = 3 \times 0$ 0.6 - 0.3 = c = 0.3 $\log_{10} y =$	n of Straight Lin mX + c $3 \log_{10} x + c$ d y intercept lies on the line 0.1 + c = c $3 \log_{10} x + 0.$	<u>ne</u> 3	Equation of Power Function $log_{10} y = 3 log_{10} x + 0.3 t$ $log_{10} y = 3 log_{10} x + 0.3 t$ $log_{10} (?) = 0.3 t$ $(?) = 10^{0.3}$ $log_{10} y = log_{10} x^3 + log_{10} (10^{0.3})$ $log_{10} y = log_{10} x^3 + log_{10} (2)$ $log_{10} y = log_{10} (x^3 x 2) t$ $y = 2 x^3$			
6).	x y		1 19	2 80		3 177	4 316	5 500	
	og ₁₀ x og ₁₀ y		0 1.28	Assume first a	and la	st point lie o	on line of best fit.	0.70 2.70	
log ₁₀ y (0, 1) <u>Finc</u> m =	$Equation of Star Y = mX - 1 log_{10} y = 2 log = 1 Find gradient. m = \frac{y_2 - y_1}{x_2 - x_1}= \frac{2.7 - 1.28}{0.7 - 0}log_{10} y = 2 log = 1log_{10} y = 2 log = 1$			n of Straight Lin mX + C 2 log ₁₀ x + C d y intercept lies on the line. 2 log ₁₀ x + 1.	<u>ne</u> 28	Equ $\log_{10} y =$ $\log_{10} y =$ $\log_{10} y =$ $\log_{10} y =$	ation of Power I 2 $\log_{10} x + 1$. $\log_{10} (?) =$ (?) = $\log_{10} x^2 + \log_{10} x$	$ \frac{-\text{unction}}{28} $ 1.28 10 ^{1.28} 0 (10 ^{1.28}) 10 (19.1) .1)	

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(1d.p.)

 $=\frac{1.42}{0.7}=2.0$

 $y = 19.1 x^2$

handheld answer 19.2 x²

7).	Х	10	20		30	40	50	
	у	20	32.6		43.3	52.9	61.8	
log	g ₁₀ x	1	A				1.70	
log	g ₁₀ y	1.3	Assume first an	id last	t point lie or	n line of best fit.	1.79	
log ₁₀ y	Iog10 y Equation of Straight Line			Equation of Power Function				
•	- (1.7 ,	$^{1.79}$ Y =	mX + c		log ₁₀ y =	0.7 log ₁₀ x +	0.6	
	(1 , 1.3)	log ₁₀ y =	$\log_{10} y = 0.7 \log_{10} x + c$ Find y intercept			log ₁₀ (?) =	0.6	
E		Fir				(?) = 10 ^{0.6}		
Find	<u>i gradien</u>	<u>. </u>	es on the line.		$\log_{10} y = \log_{10} x^{0.7} + \log_{10} (10^{0.6})$			
<i>m</i> =	$y_2 - y_1$	1.3 = 0.7	x 1 +c			. 07 .	<i>(</i>	
	$x_2 - x_1$	1.3 – 0 7	= c		$\log_{10} y =$	$\log_{10} x^{0.7} + lc$	9 ₁₀ (4.0)	
=	<u>1.79 – 1</u>	$\frac{1.3}{1}$ c = 0.6			$\log_{10} y = \log_{10} (x^{0.7} x 4)$			
=	$\begin{vmatrix} 1.7 - 1 \\ = \frac{0.49}{0.7} = 0.7 \end{vmatrix}$ log		0.7 log ₁₀ x +	0.6	$y = 4 x^{0.7}$			

8).	Х	1	1.5	2	3	4		
- /-	У	2.50	8.42	20	67.50	160		
log	J ₁₀ X	0	0.60					
log] 10 y	0.40	Assume first and last point lie on line of best fit. 2.2					

	Equation of Straight Line	Equation of Power Function
(0.6 , 2.2)	Y = mX + c	$\log_{10} y = 3 \log_{10} x + 0.4$
(0, 0.4)	$\log_{10} y = 3 \log_{10} x + c$	$\log_{10}(?) = 0.4$
	Find y intercept	$(?) = 10^{0.4}$
	(0, 0.4) lies on the line.	$\log_{10} y = \log_{10} x^3 + \log_{10} (10^{0.4})$
$m = \frac{y_2 - y_1}{x_2 - x_1}$	c = 0.4	$\log_{10} y = \log_{10} x^3 + \log_{10} (2.5)$
$=\frac{2.2-0.4}{0.6-0}$	$\log_{10} y = 3 \log_{10} x + 0.4$	$\log_{10} y = \log_{10} (x^3 \times 2.5)$
$=\frac{1.8}{0.6}=3$		$y = 2.5 x^3$

9)	X		1.2	3.1	4	4.2	5.5	6.5
0).	у		3.94	16.37	2	5.80	38.70	49.70
	log ₁₀ x 0.08 log ₁₀ y 0.60			Assume first and last point lie on line of best fit.			0.81 1.70	
log ₁₀ y	log ₁₀ y Equatio			on of Straight Li	ne	Eq	uation of Powe	r Function
	Y = 0.03 (0.08 , 0.6) $V = 0.03$ $V = 0.03$ $V = 0.03$			mX + c 1.5 log ₁₀ x +	mX + c $\log_{10} y = 1.5 \log_{10} x + 0.48$ $\log_{10} x + c$ $\log_{10} (?) = 0.48$			+ 0.48
<u>Finc</u> <i>m</i> =	$ \begin{array}{c} \hline log_{10} x \\ \hline Find gradient. \\ m = \frac{y_2 - y_1}{x_2 - x_1} \\ = \frac{1.7 - 0.6}{0.81 - 0.08} \\ = \frac{1.1}{1 - 1} = 1.5 \end{array} $			nd y intercept 6) lies on the lin x 0.08 + c	e.	$(?) = 10^{0.48}$ $\log_{10} y = \log_{10} x^{1.5} + \log_{10} (10^{0.48})$		
=				2 = c 1.5 log ₁₀ x +	= c 1.5 log ₁₀ x + 0.48		$= \log_{10} x^{1.5} + \log_{10} (x^{1.5} x)^{1.5} = 3 x^{1.5}$	log₁₀ (3.0) 3)
	0.73 (10	i.p.)						

10).	X	14.1	28.2	6	3.1	126		
- /	у	15.90	6.31	3	.16	1.58		
lo lo	log ₁₀ x log ₁₀ y		Assume first and last p on line of best fi		point lie it.	2.10 0.20		
log ₁₀ y	.15 , 1.2)	Equation Y =	on of Straight Li • mX + c	ine	<u>Eq</u> log₁₀ y	uation of Powe = -1_1 log ₁₀ x	r Function + 2.35	
(2.1	1 , 0.2) lo	log ₁₀ y =	$\log_{10} y = -1 \ 1 \ \log_{10} x + c$			log ₁₀ (?) (?)	= 2.35 = 10 ^{2.35}	
Find	d gradient	<u>.</u> (1.15 , 1.2	(1.15, 1.2) lies on the line. 1.2 = -1 x 1.15 + c 1.2 + 1.15 = c			$\log_{10} y = \log_{10} x^{-1.1} + \log_{10} (10^{2.35})$ $\log_{10} y = \log_{10} x^{-1.1} + \log_{10} (224)$		
m	$y = \frac{y_2 - y_2}{x_2 - x_2}$	$\begin{array}{c c c c c c c c c c c c c c c c c c c $						
	$= \frac{1.2 - 0.2}{1.15 - 2.1}$ $= \frac{1}{-0.95} = -1.1$ (1d.p.)		c = 2.35 $\log_{10} y = -1 \ 1\log_{10} x + 2.35$ handheld answer -1.03 $\log_{10} + 2.35$		$\log_{10} y = \log_{10} (x^{-1.1} x \ 224)$		224)	
=					$y = 224 x^{-1.1}$ handheld answer 224 x ^{-1.03}			

EXPONENTIAL LAW WORKSHEET SOLUTIONS

Example 1

TI-*nspire*

t	1		1.5	2.	2		2.5	3
V	6	5	8.5	13	.8		16.9	24
t	1		1.5	2.	2		2.5	3
log ₁₀ V	0.7	78	0.93	1.1	4		1.23	1.38
$\frac{\log_{10} V}{10^{10} V}$ $\frac{100^{10} V}{10^{10} V}$ $find grad$ $m = \frac{y_2 - 1}{x_2 - 1}$ $= \frac{1.38}{3}$ $= \frac{0.6}{2}$	$ \begin{array}{c} 0.7 \\ \hline \hline $	Equi Equi log ₁₀ (1, 0.1 (1, 0.1 0.78 = 0.78 - c = 0.4 log ₁₀	0.93 ation of Straight V = mX + c $V = 0.3 t + c$ Find y intercep 78) lies on the lie 0.3 x 1 + c 0.3 = c 18 V = 0.3 t + 0.4	<u>1.1</u> <u>t Line</u> <u>t</u> ine.		Eq g ₁₀ \ log g ₁₀ \ g ₁₀ \ g ₁₀ \ g ₁₀ \ g ₁₀ \	1.23 $uation of Expo$ $/ = 0.3 t + 0.48$ $(?) = 0.3$ $(?) = 10^{0.3}$ $/ = \log_{10} (10^{0.3})$ $/ = \log_{10} (2.0) t$ $/ = t \log_{10} (2.0)^{t}$ $/ = \log_{10} (2.0)^{t}$	1.38 nential Function $log_{10}(?) = 0.48$ $(?) = 10^{0.48}$ $(?) = 10^{0.48}$ $(10^{0.48})$ $+ log_{10} (10^{0.48})$ $+ log_{10} (3.0)$ $+ log_{10} (3.0)$ $+ log_{10} (3.0)$ $x 3.0)$
					V = 3.0 (2)	2.0) ^t		

Examples 2 to 7

For each example:

(i) show that the formula connecting y and x is of the form $y = a.b^x$ (on page 2 of handheld).

(ii) find the value of a and b, and state the formula that connects x and y.

Check the equation of the straight line (page 3) and the exponential function (page 4) on the handheld.

		1		2	3		4	5
2).		1	2	48	10	2	768	3072
	y	1	2	40	13	2	700	5072
	Х	1						5
	log ₁₀ y	1.()8	Assume first and last point lie on line of best fit.				3.49
	log ₁₀ y Equal (5, 3.49) Y (1, 1.08) Iog ₁₀ Y			ation of Straigh f = mX + c	tion of Straight Line Equation of Ex = $mX + c$ $\log_{10} y = 0.60 x + c$			nential Function 8
				y = 0.60 x + c	; t	lo	$g_{10}(?) = 0.60$ (?) = 10 ^{0.60}	$log_{10}(?) = 0.48$ (?) = 10 ^{0.48}
	Find grad	lient.	(1 1	(1) (1) (1) (1) (1) (1) (1) (1) (1) (1) (1)	ino	$log_{10} y = log_{10} (10^{0.60}) x + log_{10} (10^{0.48})$ $log_{10} y = log_{10} (4.0) x + log_{10} (3.0)$		
	$m = \frac{y_2 - y_2}{x_2 - x_2}$	<u>y1</u> X1	1.08 = 1.08 –	• 0.60 x 1 +c • 0.6 = c	ine.			
	= 3.49 - 1.08 c = 0.4		48		$\log_{10} y = x \log_{10} (4.0) + \log_{10} (3.0)$			
	$5 = \frac{2.41}{4}$	$5-1 \\ = \frac{2.41}{4} = 0.60 \\ (2d_{P}) \\ \log_{10} y = 0.60 \\ x + 0.48 \\ \log_{10} y = 0.60 \\ \log_{10} y = 0$).48	$\log_{10} y = \log_{10} (4.0)^{x} + \log_{10} (3.0)$			
	·	(Zu.p.)					y = 3.0 (4)	.0) ^x

3). X	0.5	1.2	3.8	4.1
	у у	1.79	1.53	0.86	0.80
		·			
	х	0.5	Assume first and last point lie		4.1
	log ₁₀ y	0.25	on line o	-0.10	

log ₁₀ y	Equation of Straight Line	Equation of Exponential Function				
(0.5, 0.25)	Y = mX + c	log ₁₀ y = -0.10 x + 0.30				
(4.1 , -0.10) ×	$\log_{10} y = -0.10 x + c$	$\log_{10}(?) = -0.10 \qquad \log_{10}(?) = 0.30$ $(?) = 10^{-0.10} \qquad (?) = 10^{0.30}$				
Find gradient	Find y intercept					
$m = \frac{y_2 - y_1}{y_1}$	(0.5 , 0.25) lies on the line. 0.25 = = -0.10 x 0.5 +c	$\log_{10} y = \log_{10} (10^{-0.10}) x + \log_{10} (10^{0.30})$				
$X_2 - X_1$	0.25 + 0.05 = c	$log_{10} y = log_{10} (0.8) x + log_{10} (2.0)$ $log_{10} y = x log_{10} (0.8) + log_{10} (2.0)$				
$=\frac{0.25-(-0.1)}{0.5-4.1}$	c = 0.30					
$= \frac{0.35}{0.35} = -0.10$	$\log_{10} y = -0.10 x + 0.30$	$\log_{10} y = \log_{10} (0.8)^{x.} + \log_{10} (2.0)$				
-3.6 (2d.p.)		$\log_{10} y = \log_{10} \left((0.8)^{x} \times 2.0 \right)$				
		$y = 2.0 (0.8)^{x}$				

4).	Х	2.	3	3.2	4.	6		5.0	
,	y 23.97 52.		52.70	179.52			254.80		
x 2. log ₁₀ y 1.3		3 38	Assume first and last point lie on line of best fit.		5.0 2.41]			
log	log ₁₀ y (5, 2.41) (2.3 , 1.38)		Equ	Equation of Straight Line		Equation of Exponential Function			
			Y = mX + c			log ₁₀ y = 0.38 x + 0.51			
			$\log_{10} v = 0.38 x + c$		$\log_{10}(?) = 0.38$		j ₁₀ (?) = 0.38	$\log_{10}(?) = 0.51$	
				Find y intercept			(?) = 10 ⁰		(?) = 10 ^{0.51}
Find gradient.			(2.3 ,	, 1.38) lies on the line.		$\log_{10} y = \log_{10} (10^{0.38}) x + \log_{10} (10^{0.51})$			
$m = \frac{y_2 - y_1}{y_1 - y_1}$			1.38 =	= 0.38 x 2.3 +c		$\log_{10} y = \log_{10} (2.4) x + \log_{10} (3.2)$			
	$x_2 - x_1 = \frac{2.41 - 1.38}{100}$			38 – 0.874 = c = 0.51 (2d.p.)			$\log_{10} y = x \log_{10} (2.4) + \log_{10} (3.2)$		
	5 - 2.3 = $\frac{1.03}{2.7} = 0.38$ (2d.p.)		$\log_{10} v = 0.38 x + 0.51$		$\log_{10} y = \log_{10} (2.4)^{x.} + \log_{10} (3.2)$				
			010	,		$\log_{10} y = \log_{10} ((2.4)^{x} \times 3.2)$			
							y = 3.2 (2	4) ^x	

5). x y	1.1 1.87	2.3 3.05	3.0 4.05	4.2 6.59	5.1 9.49
X	1.1	Assume first ar	5.1		
log ₁₀ y	0.27				0.98

log ₁₀ y	Equation of Straight Line	Equation of Exponential Function		
(5.1, 0.98)	Y = mX + c	log ₁₀ y = 0.18 x + 0.07		
	$\log_{10} v = 0.18 x + c$	$\log_{10}(?) = 0.18$ $\log_{10}(?) = 0.07$		
x	Find y intercept	$(?) = 10^{0.18} \qquad (?) = 10^{0.07}$		
Find gradient.	(1.1 , 0.27) lies on the line.	$log_{10} y = log_{10} (10^{0.18}) x + log_{10} (10^{0.07})$ $log_{10} y = log_{10} (1.5) x + log_{10} (1.2)$ $log_{10} y = x log_{10} (1.5) + log_{10} (1.2)$		
$m = \frac{y_2 - y_1}{y_2 - y_1}$	0.27 = 0.18 x 1.1 +c			
$x_2 - x_1$	0.27 - 0.198 = c			
$=\frac{0.98-0.27}{51-11}$	c = 0.07 (2d.p.) $\log_{10} y = 0.18 x + 0.07$			
$= \frac{0.71}{0.71} = 0.18$		$\log_{10} y = \log_{10} (1.5)^{x.} + \log_{10} (1.2)$		
4 (2d.p.)	handheld gives y intercept as 0.08	$\log_{10} y = \log_{10} \left((1.5)^{x} \times 1.2 \right)$		
		$y = 1.2 (1.5)^{x}$		

6). X	0.8 0.84	1.3 1.15		2.6 2.65	3.7 5.37		
x log ₁₀ y	0.8 -0.08	Assume first and la on line of be		point lie fit	3.7 0.73		
$\begin{bmatrix} \log_{10} y \\ 3.7, 0.8 \\ 0.8, -0.08 \end{bmatrix} = \begin{bmatrix} x \\ 0.8 \\ 0$	$\begin{array}{c c} Equation \\ \hline Find \\ \hline \\ \hline$	of Straight Line mX + C $0.28 \times + C$ y intercept) lies on the line $8 \times 0.8 + C$ 24 = C 2d = C 2d.p.) $0.28 \times - 0.30$	e. Ic	Equation of Exponential Function $log_{10} y = 0.28 \times -0.30$ $log_{10} (?) = 0.28$ $(?) = 10^{-0.28}$ $log_{10} (?) = -0.3$ $(?) = 10^{-0.3}$ $log_{10} y = log_{10} (10^{-0.28}) \times + log_{10} (10^{-0.3})$ $log_{10} y = log_{10} (1.9) \times + log_{10} (0.5)$ $log_{10} y = log_{10} (1.9)^{\times} + log_{10} (0.5)$ $log_{10} y = log_{10} (1.9)^{\times} + log_{10} (0.5)$ $log_{10} y = log_{10} (1.9)^{\times} + log_{10} (0.5)$			
(20.	p.)			y =	0.5 (1	.9) ^x	

7). X	2.0	3.1	3.8	4.4	5.1
ý y	0.53	0.24	0.15	0.10	0.06
х	2.0		5.1		
log ₁₀ y	-0.28	Assume first and last point lie on line of best fit.			-1.22

log ₁₀ y	Equation of Straight Line	Equation of Exponential Function				
(2,-0.28) ×	Y = mX + c	$\log_{10} y = -0.30 x + 0.32$				
	log ₁₀ y = -0.30 x + c	$\log_{10}(?) = -0.30$ $\log_{10}(?) = 0.32$				
(5.1, -1.22)	Find v intercept	$(?) = 10^{-0.3} \qquad (?) = 10^{0.32}$				
Find gradient.	(2.0 - 0.28) lies on the line	$\log_{10} y = \log_{10} (10^{-0.3}) x + \log_{10} (10^{-0.32})$				
$m = \frac{y_2 - y_1}{y_2 - y_1}$	$-0.28 = -0.30 \times 2 + c$	$\log_{10} y = \log_{10} (0.5) x + \log_{10} (2.1)$				
$x_2 - x_1$	-0.28 + 0.6 = c c = 0.32	$\log_{10} y = x \log_{10} (0.5) + \log_{10} (2.1)$				
$= \frac{-0.28 - (-1.22)}{2 - 5.1}$		$\log_{10} y = \log_{10} (0.5)^{x} + \log_{10} (2.1)$				
= 0.94 = -0.30	$\log_{10} y = -0.30 x + 0.32$					
-3.1 (2d.p.)	handheld gives intercept as 0.33	$\log_{10} y = \log_{10} \left((0.5)^{x} \times 2.1 \right)$				
		$y = 2.1 (0.5)^{x}$				