Basic Skills and Technology
- not a Contradiction, but a Completion

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Abstract

Are certain basic - manipulating and other - skills still necessary in math teaching? If so, then we should talk about which ones and in which amount. Some examples shall demonstrate that we can use modern computer and hand held technologies to encourage pupils and students of all levels to practise those skills which might be neglected in many cases using technology in math education. The examples presented have proved to be successful in classes but certainly there are many possibilities to improve the basic ideas in many ways using the features of all resources available at the moment.

At almost all discussions on the use of CAS in teaching mathematics the question appears mathematical basic (manipulating) skills are lost by using computers or not. At the same occasions it is emphasized by the members of the discussion that there is a need in investigating which basic skills are still necessary and in which amount.

I start from my point of view that - at this moment - we cannot and should not do without certain basic skills in mathematics. I feel unable to give any forecast for the future. I am sure that we could significantly rise the acceptance of technology in teaching mathematics by sceptic and critical teachers or technology refusing edu
cational systems if we would be able to apply technology not only to present real life problems or new - for some people radical - didactical approaches but also to improve basic calculating and manipulating techniques and mathematical capabilities. We can very consciously practise all those techniques using technology which could be neglected by a (too?) intensive use of the technology.

It would be necessary and helpful to set up a list of such basic skills which teachers would like to have practised by their students. From my own experience I can say that it is very useful to provide tools for the pupils - or students which enable them to help themselves in case of troubles and difficulties. It is not sufficient to have a textbook with lots of examples followed by the solutions and the students don´t find the way from the problem to the solution. At the other hand it is not sufficient to have problems followed by a step by step solution. Sometimes it could be useful to work out a student - computer interactive strategy to help overcoming some deficiencies.

I collected some skills which could be practised and improved using computers (that is only a glimpse and I know that the discussion could lead to the conclusion: "we don´t need this or that any longer in computer age".).

- Estimating numerical results
- Working with percentages
- Working with fractions
- Elementary algebraic operations (factorising, ...)
- Recognising types of functions by their graphs
- Functions (Relations) and their various representations
- Solving equations (linear, quadratics and systems of equations)
- Calculus techniques
- Improving 2D and 3D imagination
- Calculating mentally
- Transformations of functions
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-
-

Incited by ideas from Jan Vermeylen (Belgium), Heinz Rainer Geyer (Germany) and Johann Wiesenbauer (Austria) I tried to produce training- and practise programs for my students (secondary level II), which I want to present as seed for further developments. The platforms are DERIVE and the hand held revolution, TI-92/89. The only reason for this choi ce is because I mainly use those to CAS. I am sure that you can use each of the available systems. The goal should remain the same: developing software products which will meet our - the teachers´ and the students´- very special wishes, ideas and visions.

I have tried to work with such tools since long. In my first years working with computers I produced programs written in BASIC. It is quite nice to follow the development of hard- and software on one application: The Rule of Vieta, presenting random generated examples.

I will allow a nostalgic look into the program code – I am happy to have done this a couple of years ago. It helps a lot to write programs with modern software tools.


```
         V I E T A  -  ÜBUNGSPROGRAMM
         ----------------------------

Aufgabe 2                     davon richtig: 1


q² - 9q + 20 = 0


Wie heißt die 1. Lösung q  = ? 5
                        1
Wie heißt die 2. Losung q  = ?
                        2
leider falsch !
```

```
 Datei   Bearbeiten   Ansicht   Suchen   Ausführen   Debug   Optionen        Hilfe
                                 VIETA.BAS
 10 '**** Trainingsprogramm zum Vieta'schen Wurzelsatz
 15 '*** Programmname M 1
 20 '       Böhm 1985
 25 COLOR 15, 1
 30 RANDOMIZE TIMER
 100 CLS : LOCATE 5, 20: PRINT "V I E T A  -  ÜBUNGSPROGRAMM"
 110 LOCATE 6, 20:       PRINT "----------------------------"
 115 AU = AU + 1
 120 LOCATE 8, 10: PRINT "Aufgabe"; AU
 130 LOCATE 8, 40: PRINT "davon richtig:"; R
 135 FOR Z = 14 TO 22: LOCATE Z, 1: PRINT STRING$(75, " "): NEXT Z
 140 T = 0
 200 B = INT(RND(1) * 26) + 97
 210 X1 = INT(RND(1) * 21) - 10: X2 = INT(RND(1) * 31) - 15
 215 IF X1 * X2 = 0 OR X1 = -X2 THEN 210
 220 B$ = CHR$(B)
 225 K1 = -X1 - X2: K2 = X1 * X2
 226 IF K1 < 0 THEN V1$ = "-" ELSE V1$ = "+"
                                   Direkt
 UMSCH+F1=Hilfe   F6=Fenster   F2=SUBs   F5=Ausf   F8=Schritt            00001:001
```

Then I changed to *DERIVE* working with a class in the computer lab:

```
#3:  VIE(3, x)

            2
           x  - 49 = 0

        2
#4:   | x  - 21·x + 108 = 0 |

        2
       x  - 5·x - 6 = 0

            2
           x  - 49 = 0

        2
#5:  H | x  - 21·x + 108 = 0 |

        2
       x  - 5·x - 6 = 0

     | x = 7   x = -7 |

#6:  | x = 9   x = 12 |

     | x = -1   x = 6 |

Congratulations if you are right. Now try the next five problems!

Press any key to continue_
Simp(#5)                           Free:100%              Derive Algebra
```

The idea remained the same, the tool changed and it changed once more to hand held technology. I rediscovered my old BASIC-program, converted it to the TI´s syntax and my students practised "Vieta" in the lessons, in the breaks, at home, sometimes in the train or sometimes just for fun or – how a girl told us – as a means to find her concentration before learning for other subjects.

Practise the Rule of Vieta!

Problems: 1, correct 1
$g^2 + 18 \cdot g + 80 = 0$
1. Solution:
-10
2. Solution:
-8
right
End = ESC. next = any
MAIN   DEG AUTO   FUNC 0/30   BUSY

Problems: 3, correct 3
$c^2 - 10 \cdot c - 11 = 0$
1. Solution:
-11
2. Solution:
1
sorry, false   x1 = 11, x2 = -1
End = ESC. next = any
MAIN   DEG AUTO   FUNC 0/30   BUSY

Allow me another memory of my BASIC past: a program to revise the students´ knowledge on set theory. I am still using this tool, the students like it and it doesn´t take too long time to achieve quite reasonable results.

```
A N G A B E :

G = {9,17,12,20,15,2,3,25,7,10,16,11,6,22}
A = {20,16,17,11,6,7,15,12,25,22,2,9,3,10}
B = {20,22,6,15,3,9,11,10}
C = {12,25,11,16}


Aufgabe : (A ∖ B) ∖ (A ∖ C)


(A ∖ B) ∖ (A ∖ C) = {9,17,15}

Das stimmt leider nicht.

(A ∖ B) ∖ (A ∖ C) = {16,12,25}

                                    < E N T E R >
```

```
P r o b l e m : (AUB) ∩ (AUC) ∩ (BUC)

                                    Für die Mengen nur
                                    den Buchstaben, für
    0                               die Teilmengen nur
                                    die Ziffer eingeben.

        1      4                    Teilmengen werden
                    2    B          grün gefüllt. Beleg-
                7                    te Teilmengen können
    A       6       5               durch nochmaliges An-
                                    sprechen geleert wer-
            3                       den.
                                    Die Eingabe kann ab-
            C                       geschlossen werden,
                                    wenn nur mehr dunkle
                                    Teile vorkommen.

    r i c h t i g !
                < E N T E R >       ? löst das Problem !
```

Some other examples are following:

Practise factorising (Jan Vermeylen)

#19: POL(3)

#20:
$$\begin{bmatrix} -2 \cdot x^2 - 16 \cdot x - \dfrac{158}{5} \\ 9 \cdot x^3 - 72 \cdot x^2 + 216 \cdot x \\ -6 \cdot x^3 + 84 \cdot x^2 - \dfrac{2052 \cdot x}{7} \end{bmatrix}$$

#21:
$$\begin{bmatrix} -2 \cdot \left[x + \dfrac{\sqrt{5}}{5} + 4 \right] \cdot \left[x - \dfrac{\sqrt{5}}{5} + 4 \right] \\ 9 \cdot x \cdot (x^2 - 8 \cdot x + 24) \\ -6 \cdot x \cdot \left[x + \dfrac{\sqrt{7}}{7} - 7 \right] \cdot \left[x - \dfrac{\sqrt{7}}{7} - 7 \right] \end{bmatrix}$$

#23: POL(5)

#24:
$$\begin{bmatrix} 9 \cdot x^4 - 18 \cdot x^3 + 9 \cdot x^2 \\ 9 \cdot x^2 - 27 \\ -63 \cdot x^3 - 318 \cdot x^2 + 807 \cdot x - 378 \\ 49 \cdot x^2 + 84 \cdot x + 36 \\ 36 \cdot x^2 + 96 \cdot x + 64 \end{bmatrix}$$

#25:
$$\begin{bmatrix} 9 \cdot x^2 \cdot (x - 1)^2 \\ 9 \cdot (x + \sqrt{3}) \cdot (x - \sqrt{3}) \\ 21 \cdot (x + 7) \cdot \left[\dfrac{9}{7} - x \right] \cdot (3 \cdot x - 2) \\ (7 \cdot x + 6)^2 \\ 4 \cdot (3 \cdot x + 4)^2 \end{bmatrix}$$

POL(n) produces n random number generated problems.

The students have to factorise using pencil and paper and then check their solution.

The file could easily be changed to practise special types of factorizations if necessary.

Settings of factorization modes - Rational, Complex, ... – gives a nice variety of points of view.

(This example is some years old and could be adapted to other platforms)
Then it could look like the next trainings tool (JosefBöhm). My students like it and use it:

I will demonstrate the program including error analysis and Help function. We want to practise cubics of binomials
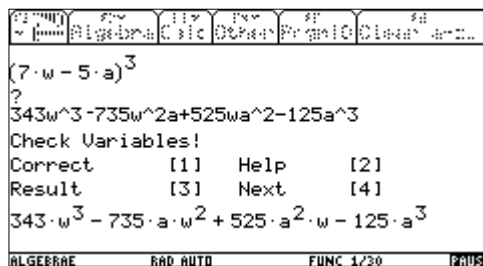

 Press 2

 Press 1

 Press 1
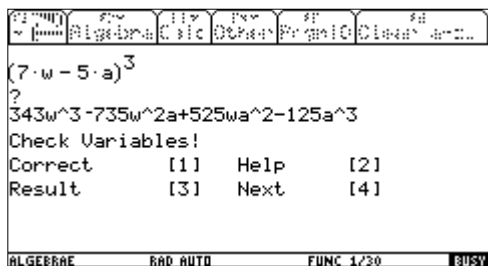
Press 3

The screen shots are self explanatory.

They practised squaring and cubing binomials, squaring trinomials, multiplying binomials mentally and with "The Quiz" they were presented a random generated sequence of problems. So you can have competitions in the class room and the pupils become very,very busy, even the weaker ones.

The next example is from **H.R.Geyer** to help students improving their **mentally calculation skills**.
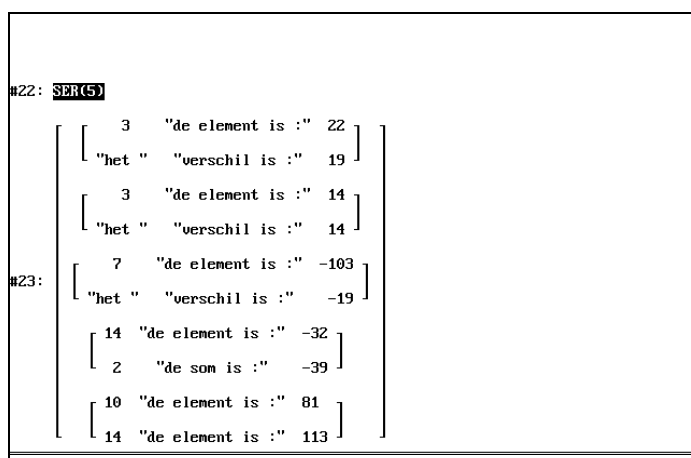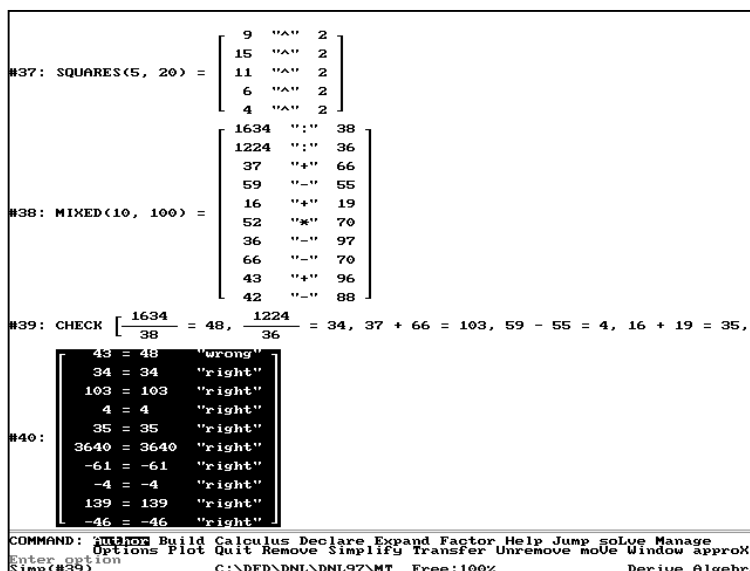
J. Vermeylen produced a *DERIVE* file to have lots of "standard problems" dealing with arithmetic series. "verschil" is Dutch, it is the difference *d* of the series.

The next screen shot is the final result of a very busy discussion in the *DERIVE* community to produce a sort of report when solving an equation and applying various equivalence transformations. **Johann Wiesenbauer** - who else - solved the problem with a very sophisticated tool.
So students are able to reproduce, what they have done using paper and pencil and check their results with the computer´s ones. They find their mistakes by themselves.


I found it useful and necessary as well that students can find the equation of a linear function, seeing its graph. They also should be able to have an imagination of the graph seeing the linear function very quick: "Match the Line!". Then I added some other tasks, as finding parallel and perpendicular lines by inspecting the grid and applying the various forms of the equation for a line
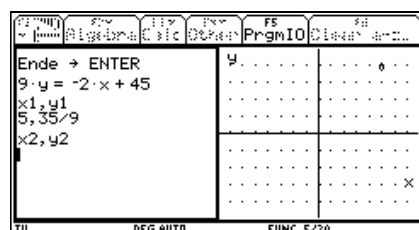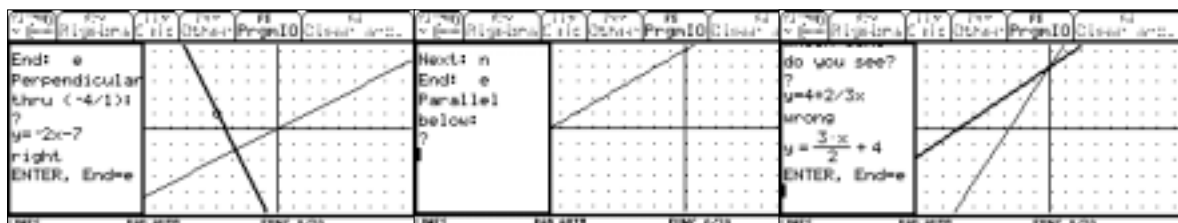
Another course of the Lines-Menu: "Find two points!"
You enter the coordinates of two points which should ly on the given line, see two "balls" on the grid and if you are lucky the line passes the two "balls". The linear function appears in various representations (explicit, implicit, ..)

Finally I produced a "game". Give some random points on the grid and let the students hit them with as few lines as possible, or do that with parabolas ......
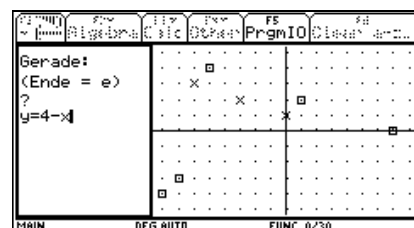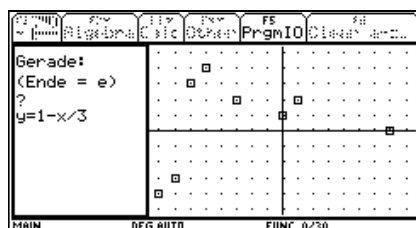
Another tool helps my calculus classes to improve their calculus techniques. I transmit the program calc() on their TI-92/89s and then they are able to practise whenever they have some time and what they believe to be necessary for them:

I know that there can be lots of improvements. It would be better to explain the way how to do it correctly to have really a learning tool. What do you say to the next program, written by Philippe Fortin from France. He calls his program "stepder" from "Stepwise deriving functions":

The last example is part of a whole sequence to teach the pupils how they can help themselves reaching a sound level in working and manipulating with fractions.

Before working with fractions we had together developed a function *kgv* to calculate the LCM of two general expressions, to check the common denominator and then we used a selfmade function *ew(expr)* to find the expanding factor for the denominator *expr* with a common denominator *gn*.

Step by step the students can perform their calculation and check it using the calculator.
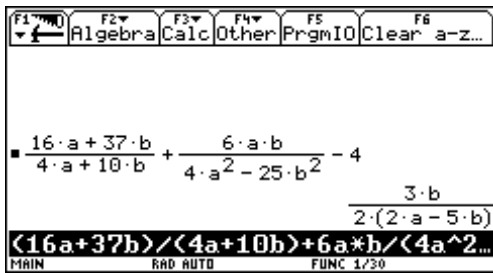
> *We have predefined a function* ***ew(expr)*** *which returns the expanding factor for a fraction with denominator* ***expr*** *and the given common denominator* ***gn***

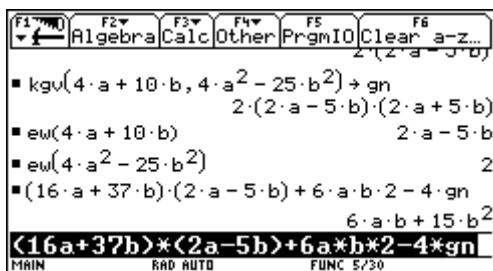# Train your skills in calculating and manipulating with fractions

Example 2: $\dfrac{16a + 37b}{4a + 10b} + \dfrac{6ab}{4a^2 - 25b^2} - 4 =$

Calculate without the *TI*. Write down your result: ....

Here it is.

Can you find a reason why this calculation could cause a very special mistake?

Compare the common denominator and the factors used to bring all the numerators over a common denominator.

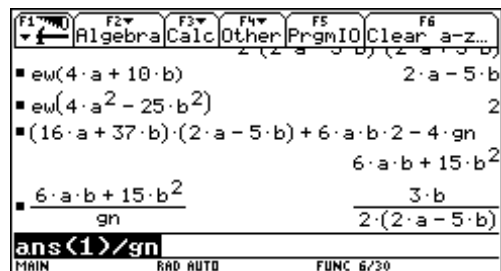Explain the last part in the edit line:

    -4 * gn:

How to work with an expanding factor
ew( )?   -4 * ew(....).

The remaining work is very easy:

Explain the relation between left and right hand side in the last row of the history area:

The underlying basic idea sounds very strange and controversary:

In addition to all the wellknown reasons to include modern technologies into mathematics teaching, which I like and support in a very high degree, I also use the technology in many cases to improve manipulatings skills which seem to be no longer of any importance by using these technologies.

If you would like to try some of the tools presented in this paper, then please contact me. I would appreciate any suggestions and ideas dealing with this topic. The TI-92/89 programs algebra(), lines() and calc() are distributed together with the *bk teachware* books (references).

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