

More TI-83, less math ?

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This article is a short written version of a presentation in Leuven, August 2001.

My answer to the above question is no - as you may have guessed. In this article examples will be given to show how to enhance mathematics when using a graphic calculator like TI83. Three main aspects of using TI-83 will be discussed:

- I TI-83 as a tool in interplay with mathematics
- II TI-83 as a tool for discovering “new” results
- III TI-83 as a tool for the teacher to produce a desire for - or at least to justify the need for - rigorous mathematical proof.

I

Graphic calculators are often accused of being a poor replacement for doing “real” mathematics. I would rather emphasize the cooperating roles of TI-83 and mathematics, as shown in example 1.

Example 1: We draw the graphs of the following functions

$$f(x) = 8x^3 + 9x^2 - 6x - 7 \text{ (fig. 1)} \quad \text{and} \quad g(x) = 8x^3 + 7.91x^2 - 8.03x - 7.88 \text{ (fig. 2)} :$$

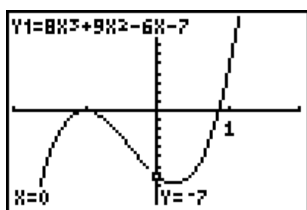


Fig. 1

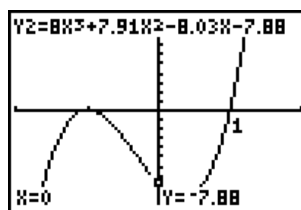


Fig. 2

The graphs are very similar. Apparently both functions have two zeros, one close to -1 and another close to $+1$. But the degree being three, there may be a third zero.

We will first examine f :

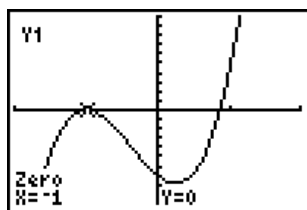


Fig. 3

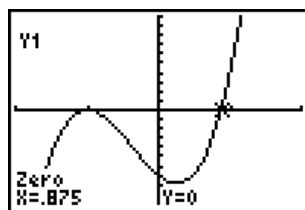


Fig. 4

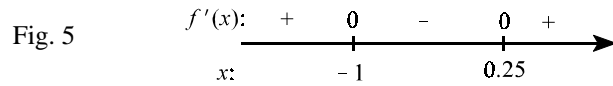
Using the TI-83 we determine two zeros: $x = -1$ and $x = 0.875$.

The derivative can tell us, whether there is one more zero:

The derivative is

$$f'(x) = 24x^2 + 18x - 6,$$

and the quadratic equation $f'(x) = 0$ has the solutions -1 and 0.25 .



The signs (fig. 5) show that the function f is increasing when $x \leq -1$, decreasing when $-1 \leq x \leq 0.25$, and increasing again when $x \geq 0.25$.

We can now conclude: $f(-1) = 0$ is a relative maximum, and $f(x)$ has no further zeros than the two zeros already found.

We now turn to the function g :

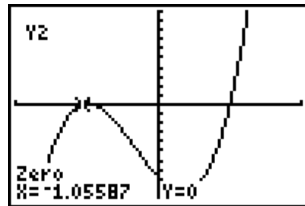


Fig. 6

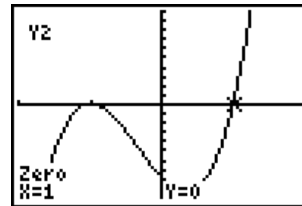


Fig. 7

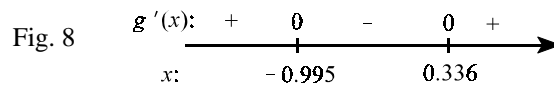
As shown in fig. 6 and fig. 7 we find find two zeros: $x = -1.05587$ and $x = 1$.

To decide whether there is one more zero we once again use the derivative.

The derivative is

$$g'(x) = 24x^2 + 15.82x - 8.03,$$

and the equation $g'(x) = 0$ has the solutions $x = -0.995$ and $x = 0.336$.



The signs (fig. 8) show that g is increasing when x passes the zero -1.05587 , because the relative maximum is not reached until $x = -0.995$. The graph passes through the x -axis again to the right of the point where $x = -0.995$. A zoom (fig. 9) can show this very clearly:

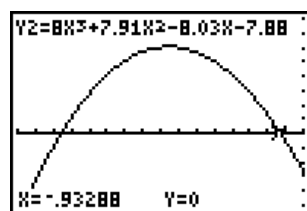


Fig. 9

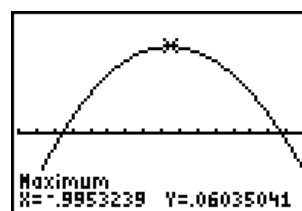


Fig. 10

Thus the function g has three zeros:

the two already found: $x = -1.05587$, $x = 1$, and the third root $x = -0.93288$.

The reason why we do not find all three zeros immediately is that two of them are very close.

Between these two we find the relative maximum: $g(-0.99532) = 0.06035$ (fig.10).

Notice how the examination of the two functions was made by a close interplay between mathematical methods and graphical methods.

II

Example 2 A classroom activity:

1. Draw the graph of the function $Y_1 = f(x) = x^3 - 3x^2 + 1$ using **ZOOM** 4: ZDecimal .
2. Choose a linear function Y_2 , so that the two graphs have three intersection points.
Students have to choose different linear functions.
3. Determine the x -values of the intersection points using [**2nd**] **CALC**: 5 **intersect** , and store the values in A, B and C .
4. Calculate A+B+C .
5. Compare the results.
6. Explain!

III

Example 3:

We study the function $f(x) = 2 - 4x + \sqrt{x}$, $x \geq 0$. The graph is shown below (fig. 11).

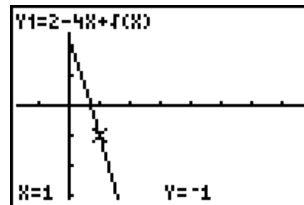


Fig. 11

The graph looks very simple, and many students would probably stop here. By zooming in we see more details (fig. 12-14).

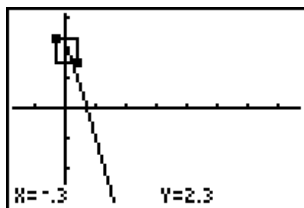


Fig. 12

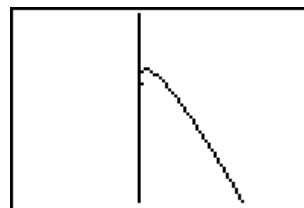


Fig. 13

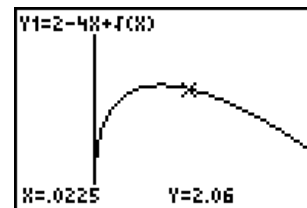


Fig. 14

Fig. 15 shows the determination of the maximum of the function:

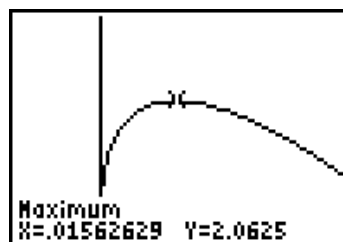


Fig. 15

Apparently the function is increasing in the interval from $x = 0$ to $x \approx 0.016$, and then decreasing.

But can we trust this graphic representation? Couldn't new details appear by more zooming in? At first (fig. 11) we could not see the tiny upturn just after zero. There may be more of those surprises hidden somewhere. A continued zooming will never give a 100% certain answer. Calculus can give a quick and final answer.

First we differentiate f :

$$f'(x) = -4 + \frac{1}{2\sqrt{x}} .$$

We then solve the equation $f'(x) = 0$:

$$f'(x) = 0 \Leftrightarrow 4 = \frac{1}{2\sqrt{x}} \Leftrightarrow \sqrt{x} = \frac{1}{8} \Leftrightarrow x = \frac{1}{64} = 0.015625 .$$

Thus f is increasing when $0 \leq x \leq \frac{1}{64}$ and decreasing when $x \geq \frac{1}{64}$, and

$$\max f(x) = f\left(\frac{1}{64}\right) = \frac{33}{16} = 2.0625 .$$

Example 4:

We want to study the function $f(x) = \frac{100\pi \sin(x) + \sin(100\pi x)}{100\pi x}$ as x tends to 0.

A graph (fig. 16) and a table (fig. 17) indicate that the limit is 1 :

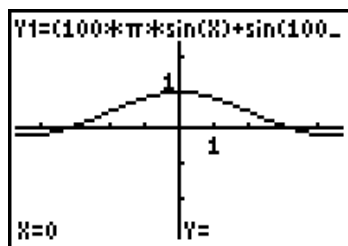


Fig. 16

X	Y1
-.4	.97355
-.3	.98507
-.2	.99335
-.1	.99833
0	ERROR
.1	.99833
.2	.99335

Fig. 17

Zooming in gives a rather confusing picture (fig. 18-19):

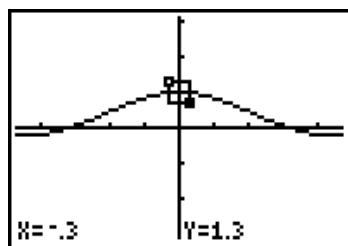


Fig. 18

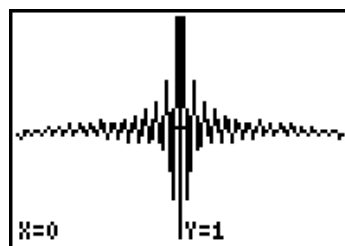


Fig. 19

Another approach is given in the following classroom activity.

Classroom activity:

1. Draw the graph of $f(x) = \frac{100\pi \sin(x) + \sin(100\pi x)}{100\pi x}$ using **ZOOM** 4: ZDecimal.

Do not forget to use **MODE** Radian.

2. What is the limit of $f(x)$ as x tends to 0 ?
3. Change the window by dividing X_{\min} and X_{\max} by 5.
Leave Y_{\min} and Y_{\max} unchanged, and press **GRAPH**.
Repeat this procedure several times.
4. What is the limit of $f(x)$ as x tends to 0 ?

Fig. 20-22 indicate that our first guess was not correct.

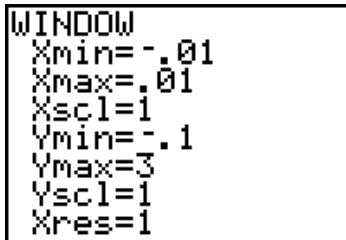


Fig. 20

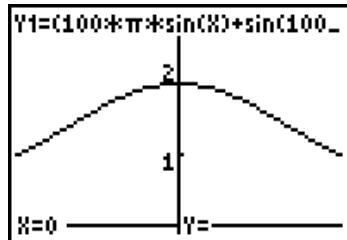


Fig. 21

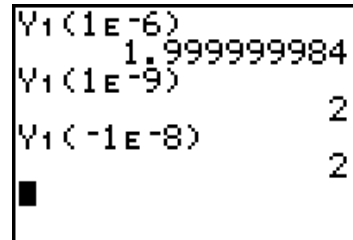


Fig. 22

**At first we believed that the limit was 1. We will now guess that the limit is 2.
But can we be certain that we have all details under control now?
A rigorous proof is needed.**