"Wings of change"
- teaching with CAS, TI-92, Graphics,...

Lecture in Oostende, 24.8.1999

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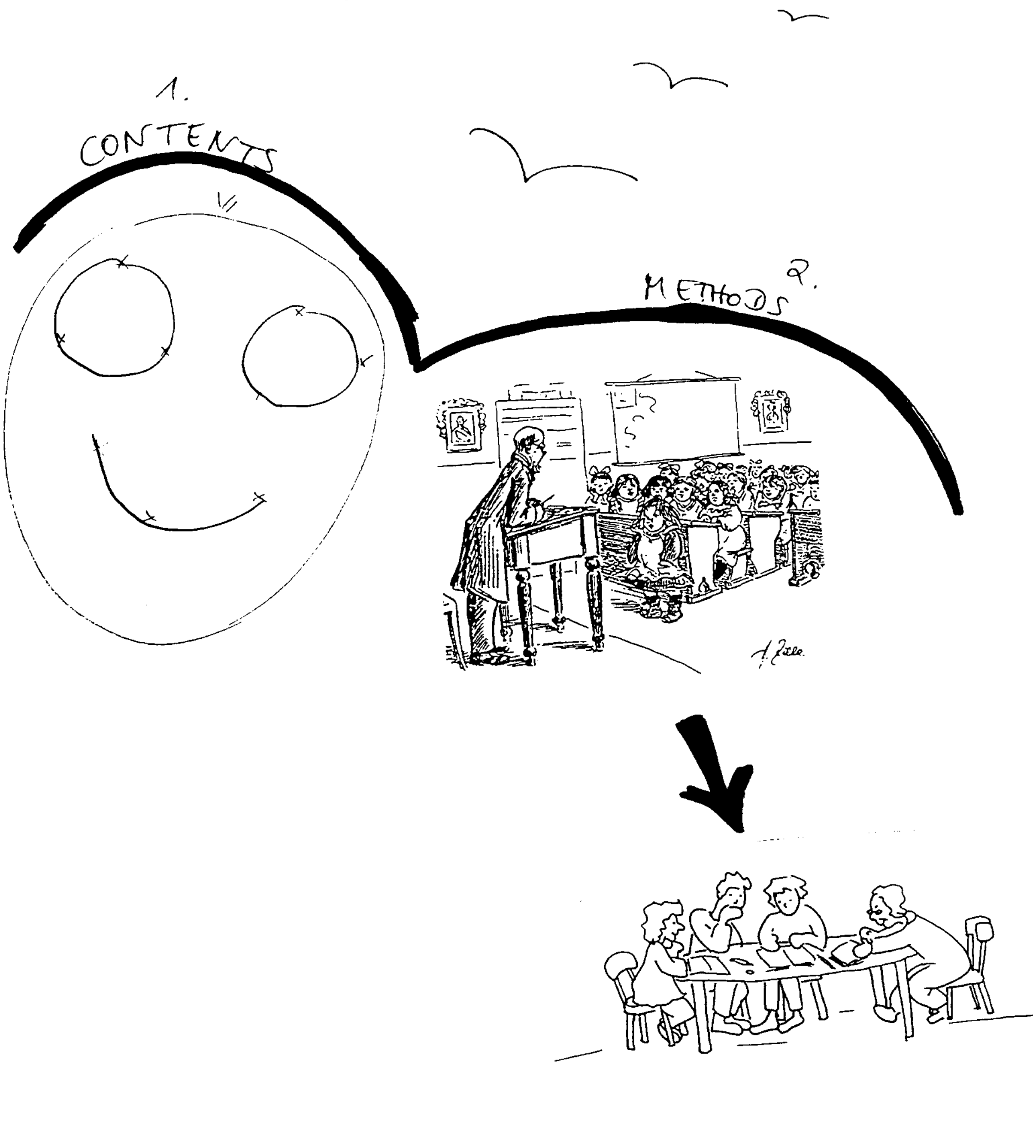
There are two main changes when teaching with the TI-92 (CAS, Graphics,....):
1. Contents: For the pupils it is easier to connect different topics, representations, subjects,.. ··· OVERVIEW
2. Methods: The new technology forces the teacher to a more pupil-centered teaching

Let these changes call the "two wings":



With several teaching examples I want to explain these two wings.

The Examples:

1. **Curve fitting**
   Here: What is the volume of the token? (see p. 3,4)

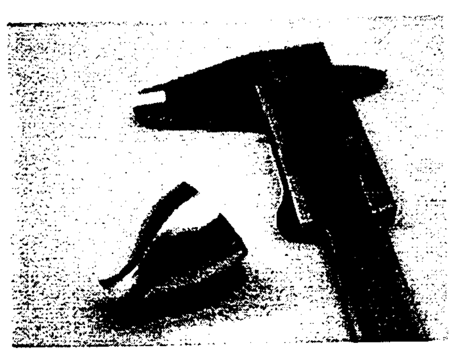
2. **Pictures with Graphs**
   See p. 5

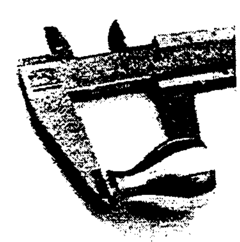
   Benefits of this kind of task:
   - Individual way/ Individual speed
   - Experimental game to come along with expressions of functions and their graphs
   - Good way to understand the meaning of parameters
   - A task which can be used in many topics and many levels

3. **Finding a generalisation**
   - "You can see a matrix" (p.6,7)
   - "What is behind Taylor?" (p.8)

4. **Structuring**
   - Modules (p.9)
   - Rational functions (p.10)

What is the volume of this figure?

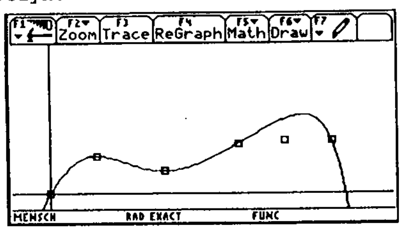




Spielstein - Möglichkeiten

1. 7 Bedingungen: 4 Punkte,
   2 als Extrempunkt festgesetzt
   1 als Wendepunkt fetsgesetzt

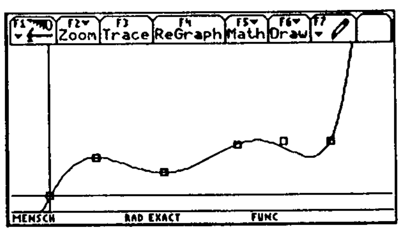
Polynom 7. Grades



V = 2,154 cm^3

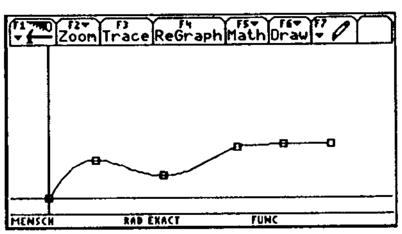
2. 6 Bedingungen: wie 1.
   nur Wendepunkt-Bedingung rausgenommen

Polynom 6. Grades



V = 1,358 cm^3

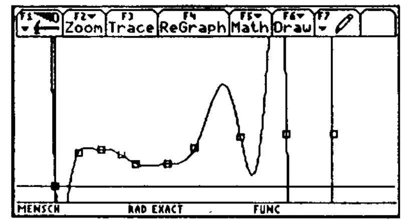
3. Aufteilung in zwei Funktionen:
   zwischen $0<x<16$ (in mm gemessen): Polynom wie 2.
   zwischen $16<x<24$: Polynpom 2. Grades



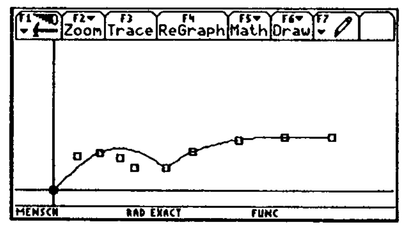
V = 1,525 cm^3

4. Bedingungen: 9 Stützpunkte
   davon 2 Extrempunkte (wie bei 1.)
   davon 1 Wendepunkt (wie bei 1.)

Polynom 12. Grades

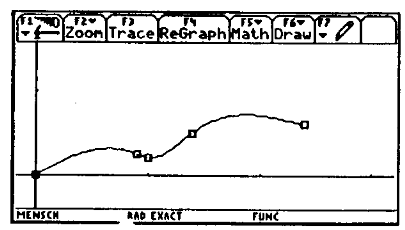


5. Splines , 5 Stützpunkte (die anderen eingezeichneten Punkte sind die restlichen Messwerte)



V = 1,728 cm^3

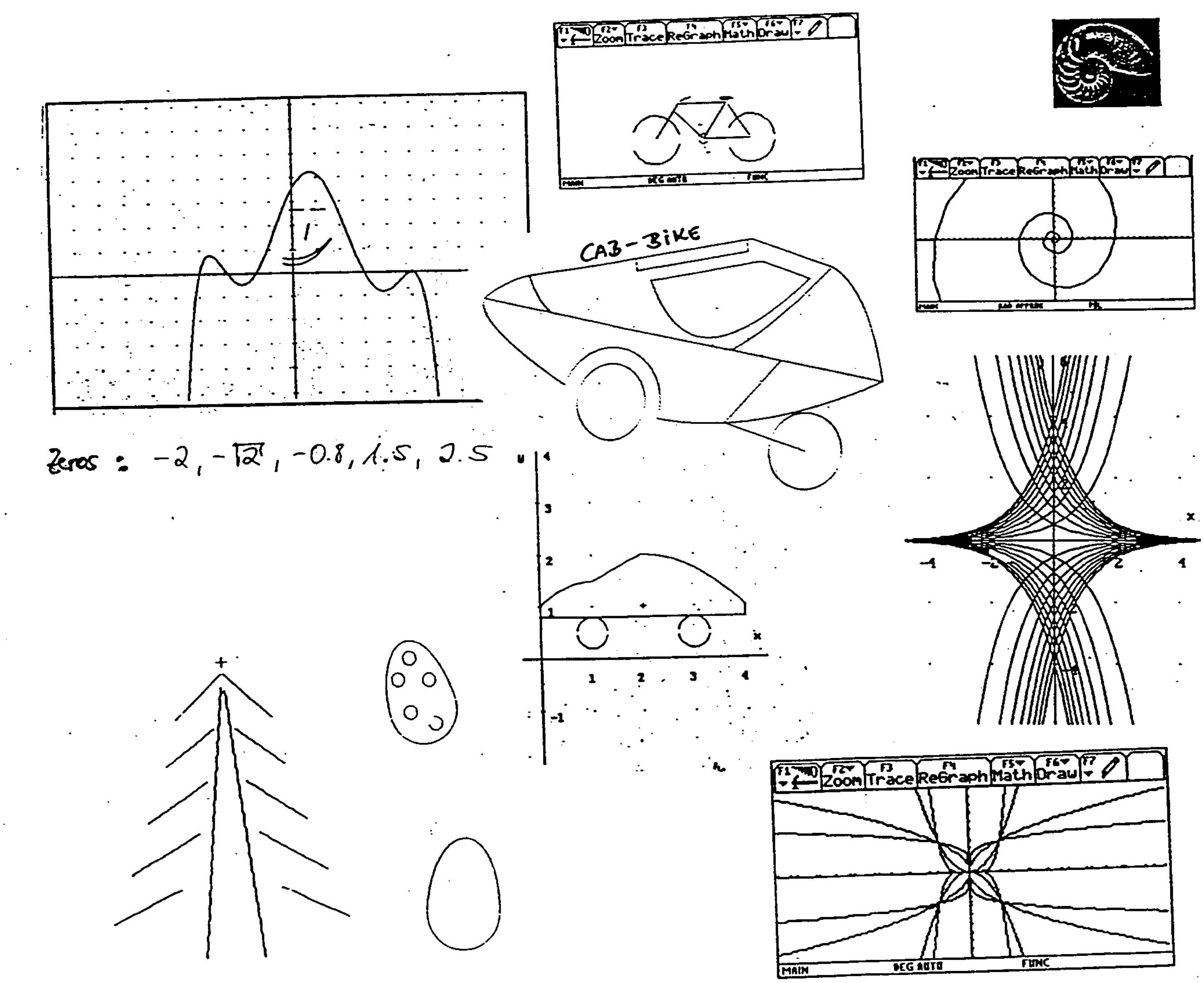
6. Splines (Werte von Jens):



V = 1,994 cm^3

Bringe das Bild auf den Bildschirm

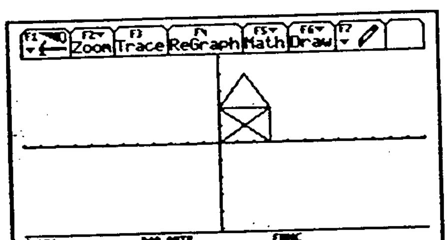
. . . .

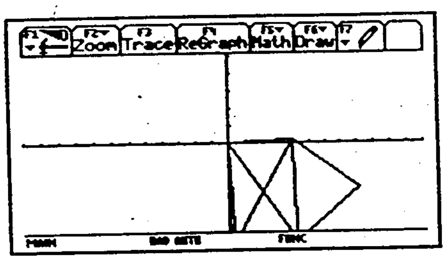


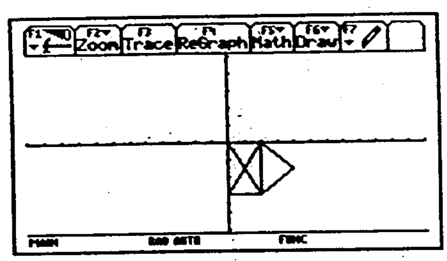
$$A = \begin{pmatrix} a & -b \\ b & a \end{pmatrix} \quad ; \quad a, b \in \mathbb{R}$$

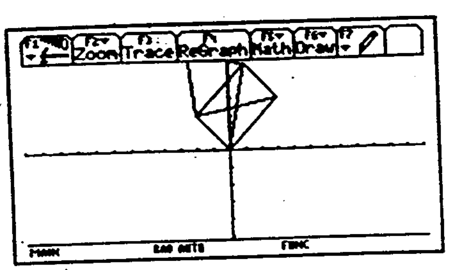
Welche elementargeometrischen Abbildungen werden durch die Matrix A beschrieben?

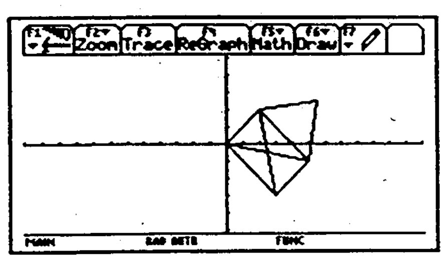
Which transformations are described by this matrix A?











A Pupils Poster:

Elementargeometrische Abbildungen durch die Matrix A

$$K \cdot \begin{pmatrix} \cos\alpha & -\sin\alpha \\ \sin\alpha & \cos\alpha \end{pmatrix} \cdot \begin{pmatrix} x \\ y \end{pmatrix}$$

α = Drehwinkel

Bedeutung von K:

$0 < K < 1$: Ortsvektor wird gestaucht

$K > 1$: " " gestreckt

$0 > K > -1$: " " gestaucht und am Nullpunkt gespiegelt

$K < -1$: Ortsvektor wird gestreckt und am Nullpunkt gespiegelt

"What is behind Taylor?"

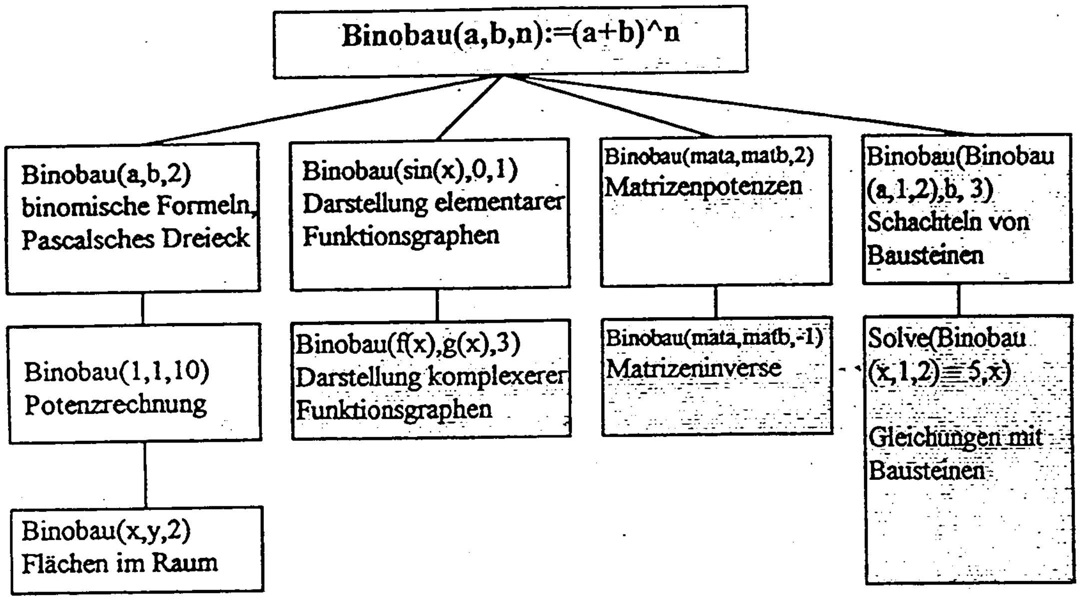
1. **Let the curves to all the expressions of the Algebra window plot out in the Graphic window.**

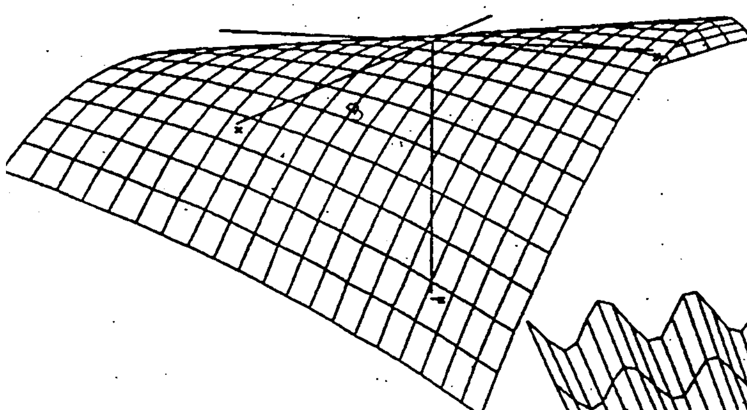
   What can you realize?

2. **Write the expressions of the Taylor expansions to the sine-, cosine- and e-function down into a table.**

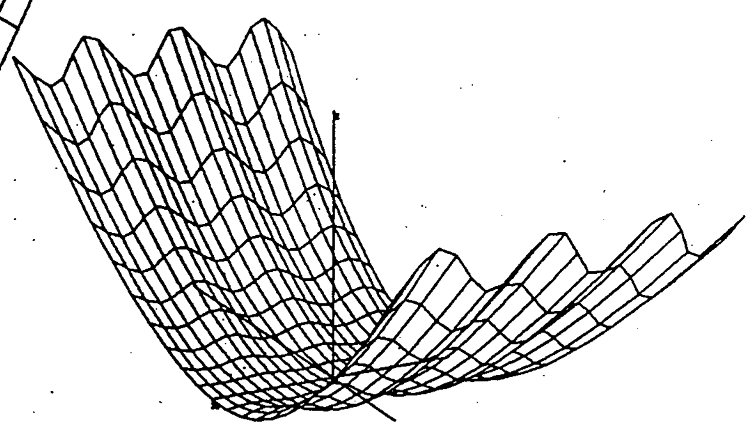
   Can you find a general rule?

Grad	sin (x)	cos (x)	e^x
1	x	1	$x+1$
2	x	$-\frac{x^2}{2}+1$	$\frac{x^2}{2}+x+1$
3	$-\frac{x^3}{6}+x$	$-\frac{x^2}{2}+1$	$\frac{x^3}{6}+\frac{x^2}{2}+x+1$
4	$-\frac{x^3}{6}+x$	$\frac{x^4}{24}-\frac{x^2}{2}+1$	$\frac{x^4}{24}+\frac{x^3}{6}+\frac{x^2}{2}+x+1$
5	$\frac{x^5}{120}-\frac{x^3}{6}+x$	$\frac{x^4}{24}-\frac{x^2}{2}+1$	$\frac{x^5}{120}+\frac{x^4}{24}+\frac{x^3}{6}+\frac{x^2}{2}+x+1$
6	$\frac{x^5}{120}-\frac{x^3}{6}+x$	$-\frac{x^6}{720}+\frac{x^4}{24}-\frac{x^2}{2}+1$	$\frac{x^6}{720}+\frac{x^5}{120}+\frac{x^4}{24}+\frac{x^3}{6}+\frac{x^2}{2}+x+1$
7	$-\frac{x^7}{5040}+\frac{x^5}{120}-\frac{x^3}{6}+x$	$-\frac{x^6}{720}+\frac{x^4}{24}-\frac{x^2}{2}+1$	$\frac{x^7}{5040}+\frac{x^6}{720}+\frac{x^5}{120}+\frac{x^4}{24}+\frac{x^3}{6}+\frac{x^2}{2}+x+1$



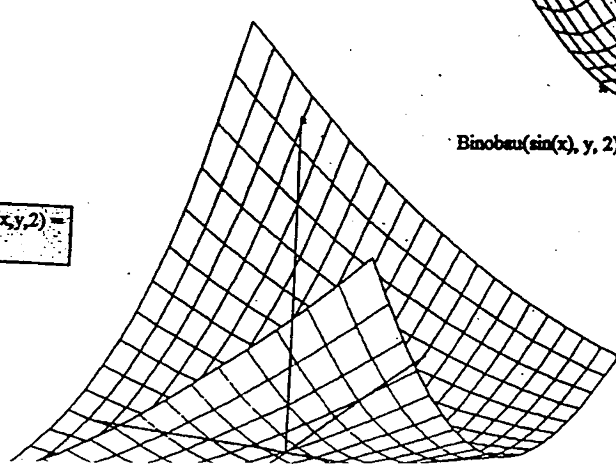


Der Aufruf -Binobau(x,y,2)



Binobau(sin(x), y, 2)

binobau(x,y,2) = (x+y)^2



Gebrochen-rationale Funktionen

Def.: Es sei $P_n(x)$ ein Polynom n-ten Grades und $Q_m(x)$ ein Polynom m-ten Grades. Dann heißt

$$\frac{P_n(x)}{Q_m(x)}$$

eine **gebrochen-rationale Funktion** oder kurz **rationale Funktionen**.

An den Nullstellen des Nennerpolynoms $Q_n(x)$ ist die Funktion nicht definiert (**Definitionslücken**).

Das Besondere an Graphen von gebrochen-rationalen Funktionen ist ihr asymptotisches Verhalten (ASYMPTOTEN - griech.: Nichtzusammenfallende) an den Definitionslücken und gegebenfalls für $x \to \infty$.

1. Definitionslücken:

Welche grundsätzlichen Unterschiede gibt es bei den Definitionslücken? Betrachte zum Beispiel die Graphen zu den folgenden Funktionen:

$$\frac{x+1}{x^2-4}\ ;\ \frac{1}{x^2+1}\ ;\ \frac{x^2-4x+2}{(x-2)^2}\ ;\ \frac{x^2-1}{x+1}$$

2. Verhalten für $x \to \infty$:

Finde die prinzipiellen Möglichkeiten für asymptotisches Verhalten für $x \to \infty$. Betrachte zum Beispiel die Graphen zu den folgenden Funktionen:

$$\frac{2}{x^2-1}\ ;\ \frac{x^5-x^4+1}{1+2x^5}\ ;\ \frac{x^3+2x^2-3x+4}{4x-4}\ ;\ \frac{x^3-2x^2-x+3}{x^2-1}$$

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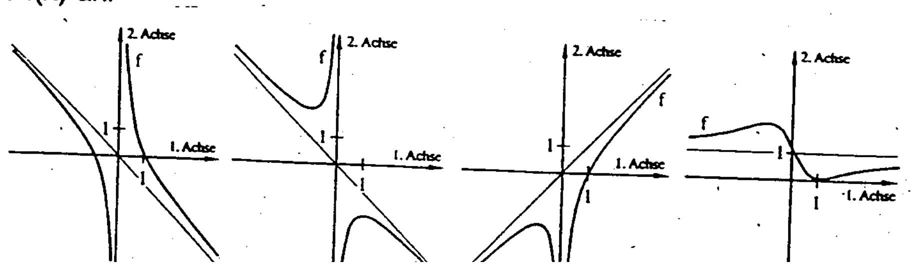
3. Gib f(x) an.



Questions, thoughts, proverbs,... A Conclusion?

Do the examples describe "Discovery learning" ?

Tell me and I will forget,
Show me and I will understand,
Involve me and I will learn.
Chinese Proverb

The Computer forces us to think about things,
we should have been thinking about for a long time.
Hans Schupp, 1994

Are you ready for takeoff - ready for a change - ready for the chance???

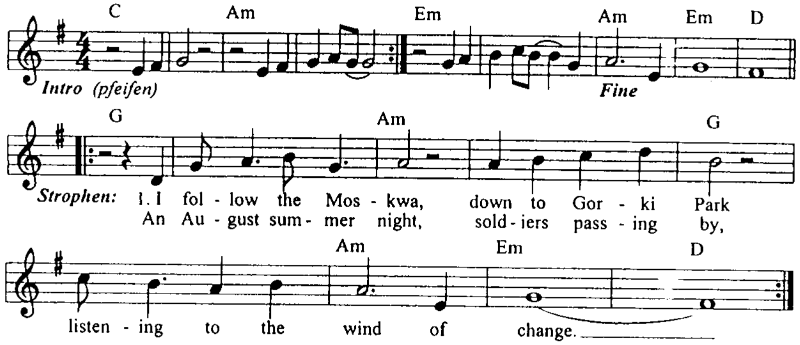
Do the "wings" bring us to heaven - with or without a connection to the earth??

* A training for chickens who have always lived in boxes!

Where is your position?

Wind of change

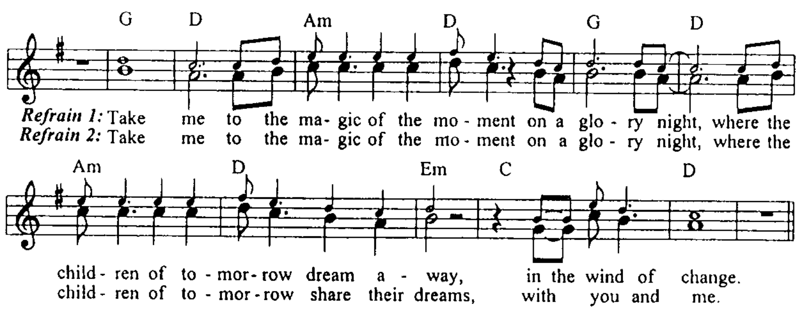
Words & Music
by Klaus Meine



Intro (pfeifen)

G *Am* *G* *Am* *Em D*
2. The world is closing in, did you ever think that we could be so close like brothers.
G *Am* *G* *Am* *Em D*
The future's in the air, I can feel it ev'ry where, blowing with the wind of change.

Refrain1:



G *Am* *G* *Am* *Em D*
3. Walking down the street, distant memories are buried in the past fore - ver.
G *Am* *G* *Am* *Em D*
I follow the Moskwa, down to Gorki Park, listening to the wind of change.

Refrain2: Take me... ...share their dreams, with you and me.
Refrain1: Take me... ...dream away, in the wind of change.

Refrain2: Take me... ...share their dreams, with you and me.
Refrain1: Take me... ...dream away, in the wind of change.

Intro bis Fine