Some variations on a quadratic theme

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0. Introduction

I have had the pleasure of working in Belgium several times in the past, and of meeting colleagues from both Belgium and the Netherlands through various T-cubed activities. So I am delighted to have been asked to contribute to this symposium. In the UK I have worked for many years to explore how new technology can help both the learner and the teacher of mathematics, and for the past 15 years this has been mainly concentrated on the use of graphing calculators, such as the TI-83, at all levels from upper primary school to university - and in teacher education. In our school system, though, the use of such technology is not compulsory (yet!) - and is fiercely resisted by some academics. There seems to be a fear about students "switching on calculators and switching off brains". So I plan to give a few examples from my own experience of how I hope we can use graphing calculators to stimulate brains! In an attempt to make this hang together I have taken a theme suggested by Guido Herweyers: that of the 2nd degree polynomial function, which we call "quadratic", and whose graph is a parabola. I will also try to slip in some ideas from applied mathematics, such as statistics and modelling, as well as from geometry!

1. Modelling mathematics

The starting point here is to use data we know are generated from a quadratic function.

Problem 1.0 <u>Putting on the brakes</u>

The first example is that of the "Shortest Stopping Distances" of a car, taken from our government's "Highway Code" for new drivers. This gives a table for the shortest stopping distance from various speeds: "on a dry road, a good car with good brakes and tyres and an alert driver will stop in the distances shown". (Notice the quaint units of measurement!)

Shortest stopping distances - in *metres* and feet

Speed: mph	Thinking distance		Braking distance		Overall stopping distance	
20	*6*	20	*6*	20	*12*	40
30	*9*	30	*14*	45	*14*	75
40	*12*	40	*24*	80	*24*	120
50	*15*	50	*38*	125	*38*	175
60	*18*	60	*55*	180	*55*	240
70	*21*	70	*75*	245	*75*	315

What is the overall stopping distance for 45 mph? Or for 100 mph?
What is the formula for the overall stopping distance in feet in terms of the speed in mph?
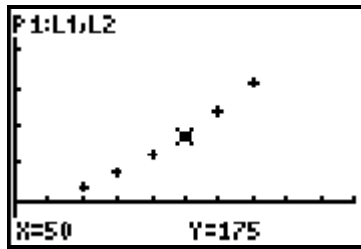
So, with the data stored in lists L_1 and L_2, say, we can draw a scatterplot from the table and try to superimpose a quadratic function, starting with the simplest one $y = x^2$.
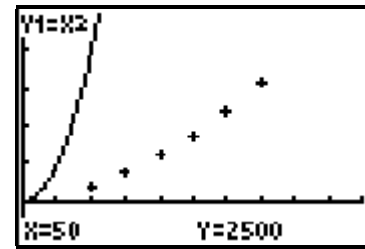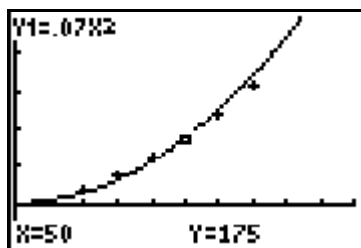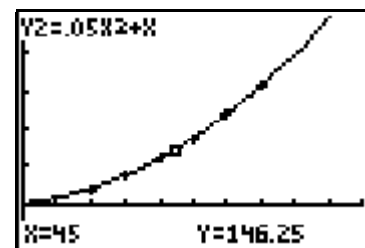

In order to make it pass through a data point such as (50,175) we need to "scale it down" by a factor of 175/2500 = 0.07 - which gives a very close, but not exact, fit. However we can see if the TI-83 can do better by using its built-in Quadratic Regression model.

Now we can validate this result from the original table since the "Thinking distance" in feet is just the same as the speed in mph, and the "Braking distance" in feet is half the square of the speed in mph. So the "Stopping distance" is just the sum of these. We can also see that the differences between the quadratic values in L_2 are in arithmetic progression - hence linear, and the second differences are constant.

Problem 1.1 : Handshakes
If there are N people in a room and each person shakes hands with each other, how many handshakes will take place?

N	2	3	4	5	6	7
S	1	3	6	..	..	..

Problem 1.2 : Regions of a circle (extending to other polynomials)
An identical problem to Problem 1.1 is to place N points round the circumference of a circle and to find the number of chords required to join each pair of points. These chords will divide the circle up into a number of regions. The problem is to find the greatest number of such regions which can be generated using N points. For N>5 you will need to space them out unequally to avoid having any places where 3 or more chords meet in a single point. (Note: the "doubling" rule soon breaks down - for which value of N ?)

N	2	3	4	5	6	7
R	2	4	8	..	..	..

2. Modelling the real world
Here we need to be able capture data from situations which we believe will be well modelled by quadratic functions.

Problem 2.0 <u>Sydney Harbour Bridge</u>
An example could be from a photograph of the span or arch of a bridge, such as that of Sydney Harbour Bridge below.

You could just draw some axes over this picture and measure a set of co-ordinates from one of the curves which resembles a parabola. Alternatively you could scan the picture into a computer (not needed if you use a digital camera, of course) and use software such as MS Paint^{TM} to read off pixel co-ordinates using the mouse - measured from the top-left corner.

X	58	152	252	348	440	533	627	725
Y	484	411	352	304	268	244	232	228
X cont	817	918	1020	1122	1236	1285	1349	
Y cont	236	255	286	330	386	420	461	

With this data entered into lists L_1 and L_2, we can start to model the image. First we need to transform co-ordinates relative to an origin at the bottom-left corner, which has pixel co-ordinates (0,721) - so we store $721 - L_2$ into L_3. Then we can re-scale the co-ordinates into units of, say, 100 pixels - and so store $L_1/100$ in L_4 and $L_3/100$ in L_5. Again we can draw the scatterplot of L_4 against L_5 and try to superimpose a quadratic function "by eye".
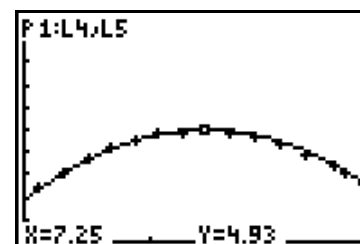
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Again, finding a quadratic regression gives a very good fit!
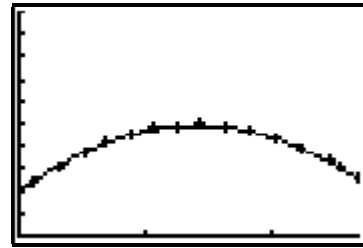
But it is also a nice exercise in algebra to equate the polynomial in the form:
$$ax^2 + bx + c \quad \text{with that in the form:} \quad p(x - q)^2 + r$$

In fact the second form is far more useful for modelling, and gives a live example of the general idea of transformations of functions f(x) in the form: $pf(qx + r) + s$ which we shall return to later. Of course the transformation from $ax^2 + bx + c$ to $p(x - q)^2 + r$ is also the fundamental step in deriving the formula for the solution of a quadratic equation!

Problem 2.1 Flexing a ruler
Another way to generate a curve which may be well fitted by a quadratic, or cubic, function is to flex a long ruler into a "U", or an "S", shape and trace over it onto graph paper. You can then take a set of co-ordinates from some sample points and try to fit suitable polynomial functions to them.
You can also try hanging a chain or heavy rope and then take a photograph with a large sheet of squared paper as background. (Note: although the theoretical curve is a "catenary", not a parabola, it should be very well fitted by a quadratic.)


3. Modelling motion
With the CBR used in conjunction with the TI-83 you have a robust, practical and inexpensive way of capturing data from moving objects. Starting with experiments involving students walking in front of the CBR you can develop the idea of a distance-time graph and its interpretation - leading to ideas of constant velocity producing straight line graphs, whose equations can be found "by eye", or using Linear Regression. Fundamental in fitting a line of the form $Y = aX + b$ to a distance-time graph is the interpretation of units. If Y is a distance measured in metres and X is a time in seconds then a must be measured in metres per second, and hence a velocity, and b is a distance in metres, and hence the displacement at time X=0. We now seek some simple, robust and non life-threatening experiments which should produce quadratic functions!


Problem 3.0 Bouncing balls
For obvious reasons it is not a good idea to try to bounce a ball on top of the CBR! So the initial experiment involves holding the CBR at a fixed height above the (hard) floor and releasing a bouncy ball from about 0.5 metres below it. The captured data (with some light smoothing) looks like a set of hanging loops (roughly parabolic) of diminishing depth. The horizontal line is where the ball has bounced out of the field of view and so we are just measuring the distance from the CBR to the floor. (Later we shall see that the RANGER program for the CBR contains an application, called "Ball bounce", to transform these measurements to give heights of the ball above the floor.)

Because the data has been captured without using a "real-time" display we can also use RANGER to show velocity-time and distance-time graphs as below.
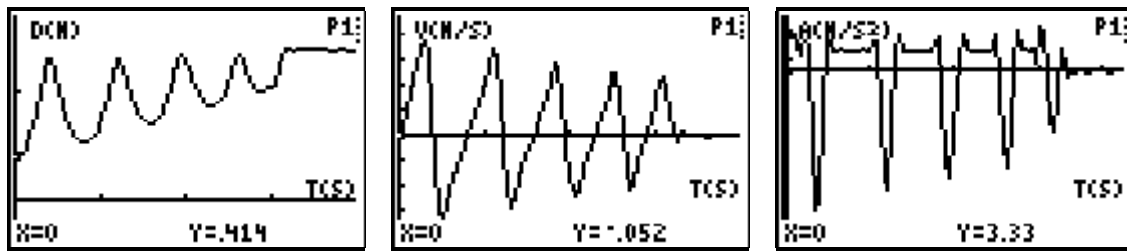
Here it would be better if the data had been plotted discretely as scattergrams rather as joined up "xy-lines" - so we just have to filter out the singularities mentally. But we can see that each arch, or loop, of the distance-time graph corresponds to a line segment on the velocity-time graph - and that these segments all look parallel. Similarly these correspond to the roughly horizontal (positive) sections of the acceleration graph. (Of course, **we** know that if the displacement is quadratic, then the velocity must be linear and the acceleration must be constant.) Now we can use RANGER's "Plot Tools" to "Select a domain" and hence extract data for one loop of the displacement-time grap. Leaving RANGER we have the time data stored in list L_1 and the distance data in list L_2. So we can now plot a scattergram from these and again fit quadratic functions by eye and using regression.
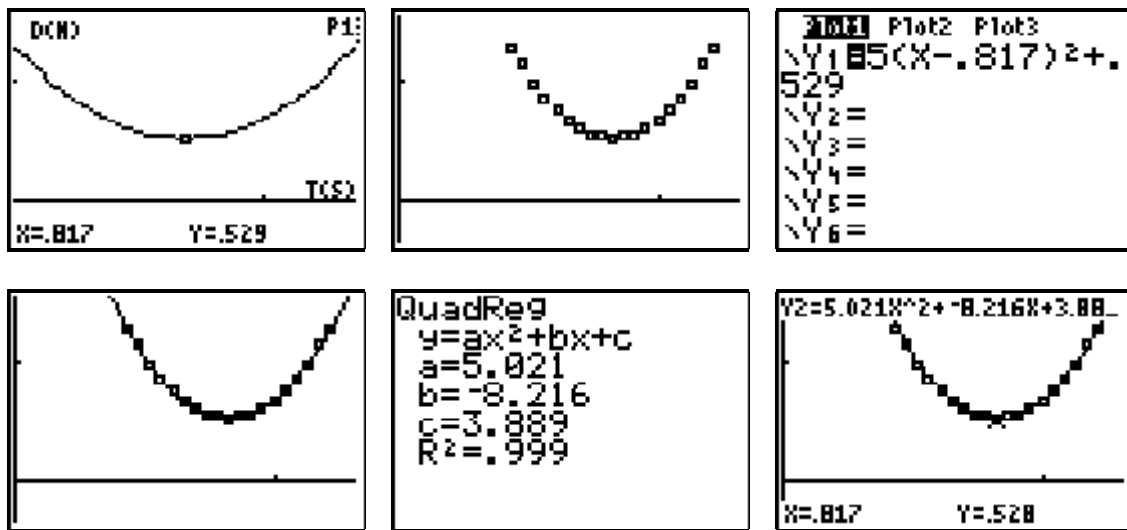
If the acceleration a is constant, then the velocity will be given by a linear function such as $v = at + b$. Hence the displacement s at time t will be given by the area under its graph between 0 and t. This is a trapezium whose area is given by the base multiplied by the average height, i.e. $s = t \, \frac{1}{2} \, (b + at + b) = \frac{1}{2} at^2 + bt$. But if the initial distance at time $t=0$ is given by c, then the distance at time t will be given by: $d = \frac{1}{2} at^2 + bt + c$. So the regression model gives a value for a of 5.021 which differs from the theoretical solution: $a = g/2$ by approximately 2.5%, which is a pretty good experimental result. (Note: Remember, too, that with d in metres and t in seconds then a will be an acceleration, b is a velocity and c is a distance.)

Problem 3.1: <u>Galileo's experiment</u>
Can you set up the CBR to measure something rolling down a gently inclined plane as a way of estimating g ?

Problem 3.2 <u>Walk a parabola!</u>
Can you walk in a straight line towards and away from the CBR in such a way that your
distance-time graph is a parabola which can be well-fitted by a quadratic function?

Problem 3.3 <u>Many bounces</u>
Here are some screens of data captured using the Ball Bounce Application from RANGER.
Some students extracted data to approximate the successive maximum heights of the
bounces. They decided to fit a quadratic function to the data, shown in the table below.
Was this a sensible thing to do?
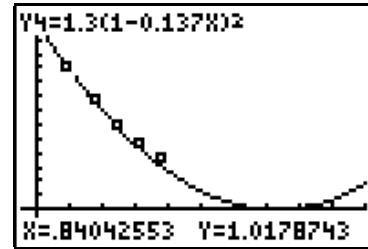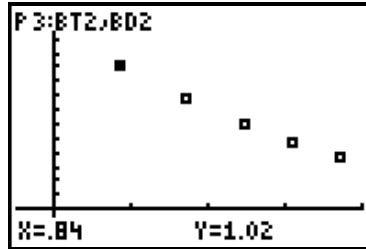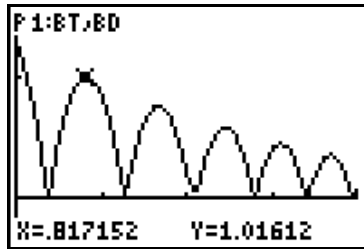
N	x (s)	y (m)
0	0.00	1.30
1	0.84	1.02
2	1.68	0.78
3	2.45	0.60
4	3.10	0.46
5	3.69	0.37

Problem 3.4: <u>Bunjee jumping and swinging</u> (for non-polynomial models)
Try setting the CBR up using the Ball Bounce Application from RANGER with other
physical experiments. E.g. suspend a mass from a spring, or piece of elastic, over the CBR
and measure some vibrations, or swing a simple pendulum in front of the CBR.


4. Products of lines
Up to this point we have used a rather "applied" approach to motivate the need for, and use
of, quadratic functions! Now we return to a rather more "pure" approach, but nonetheless
rather unconventional, to prompt a move from linear functions into quadratic functions.

Problem 4.0: <u>The product of lines</u>
The basic idea is to have a pair of simple linear functions $Y_1(x)$ and $Y_2(x)$ with their graphs
drawn in a suitable window. The investigation is one on the "arithmetic of lines". For
example can you predict the shape of the graph of the function $Y_3(x) = Y_1(x) + Y_2(x)$? If it
is a linear function, how are its gradient and y-intercept related to those of Y_1 and Y_2? The
surprise should come when we change to: $Y_3(x) = Y_1(x) * Y_2(x)$!
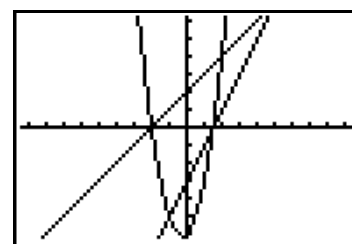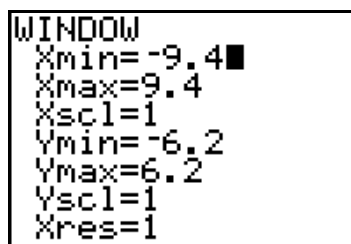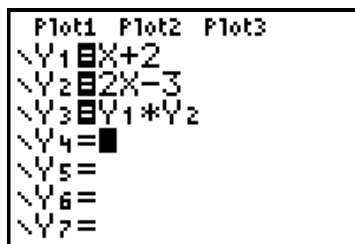
By tracing you should be able to relate the zeros of Y3 to those of Y1 and Y2, and the sign of Y3 to the signs of Y1 and Y2.
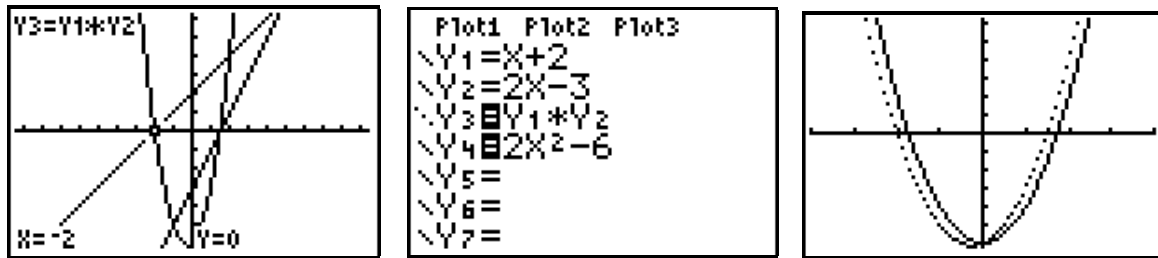

You can de-select the graphs of Y1 and Y2 being displayed, and change the style of Y3. Now you could try to write an expanded version of Y3 in Y4 as a quadratic function without brackets (or parentheses) - like the erroneous attempt above. If you can't distinguish whether the graphs are equivalent or note you might have to adjust the WINDOW. Here wis an example of using technology to reverse a conventional teaching order - instead of splitting a quadratic function into linear factors we have used a converse approach of arriving at a quadratic function by the product of two linear functions. Of course it is true that all products of linears are quadratic. Is the converse true? How could you demonstrate its truth or falseness? Of course you could extend the product of lines to produce cubics!

5. Optimisation
Since areas are measured in square metres it is likely that we can find a variety of problems concerned with maximising an area subject to some constraint. Here we use a very simple one, which can be extended to many more sophisticated examples.

Problem 5.0: <u>the greatest rectangle</u>
You have a 4 metre rope and four posts from with which you can fence out a rectangle. What shape should this rectangle be if it is to contain the maximum possible area? Well, if one side is X, then the other side will be 2-X and the area will be X(2-X).
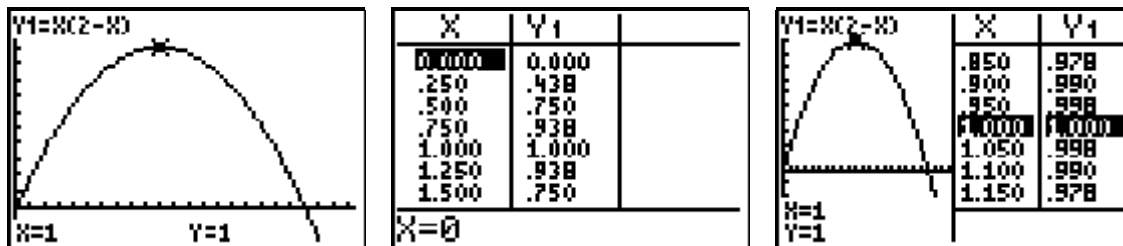
This makes a nice context for introducing the three representations: the symolic (the equation Y1 = X(2-X)), the graphic (its graph) and the numeric (a table of values). The process of seeking a maximum can be seen as zooming in round a point on a graph, or as narrowing down the tabulation in a smaller interval. The split screen shows all the representations. We shall pursue the "zooming" approach further in 9 below. Of course we have yet to interpret our solution in terms of the geometry of the problem! Here the two sides will be equal, and so the solution is the square. We can also go on to discuss the symmetry of the rectangle problem and the symmetry of the parabolic graph - also the interpretation of the zeros, why it does not make sense to have negative areas etc.

Problem 5.1: <u>find some good problems!</u>
Ideas include finding the greatest area of a figure made by a using a wall, a fixed length of rope and 1 or 2 poles - such as an isosceles triangle, a rectangle, a trapezium etc. Also volume problems such as cutting squares from the corner of a rectangular piece of card and folding to form a tray of maximum volume, cutting a sector of a circle and folding it to form a cone of maximum volume, designing a cylindrical container to hold a volume of 1 litre which has minimum volume …..

6. Transforming graphs
Another approach to quadratics is via the transformation of functions and graphs - which in the UK is usually only attempted with more able students in the 11-16 range. Yet with ICT tools this becomes accessible to a wider range of students.

Suppose we start with the function $Y_1(X) = X^2$. Then we can easily investigate transformations such as $Y_2(X) = Y_1(X) + 2$, or $Y_3(X) = Y_1(X+2)$.
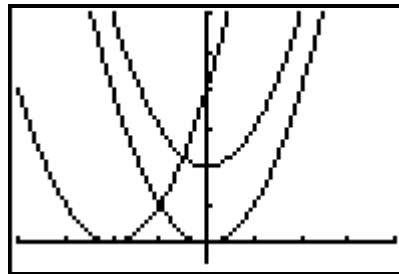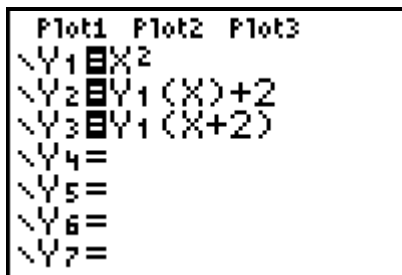

You can use TRACE to sort out which graph is which. Of course Y_1 can be any function - so the same technique can be used for older students, say, to explore the transformations of graphs of trigonometric functions, where terminology such as amplitude, phase shift, frequency etc. take on both graphical and symbolic meanings.

Problem 6.0: <u>Find the function</u>
A popular activity to do with guessing functions, and their transforms, is to have a set of screen shots taken from the graphing calculator's screen and re-scaled so that they are the physical size of the calculator's display. These images can then be printed, and then photocopied onto overhead transparencies. These can be cut up with scissors and used for a "match my graph" game. (In fact each such image can be used in four different orientations!) The next pages have a set of 16 such images which you are welcome to use, but you may prefer to make up your own set.

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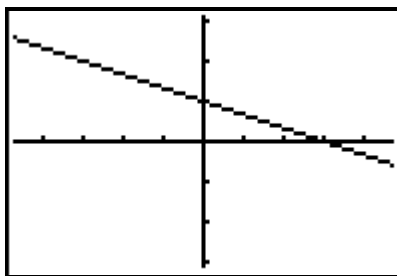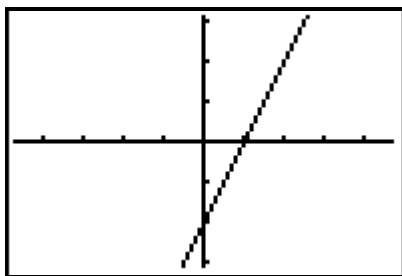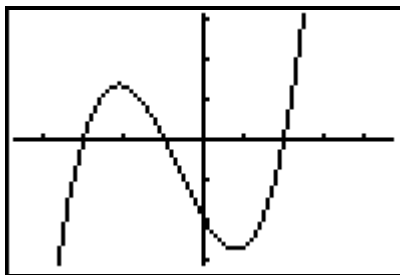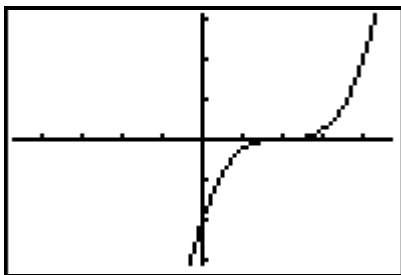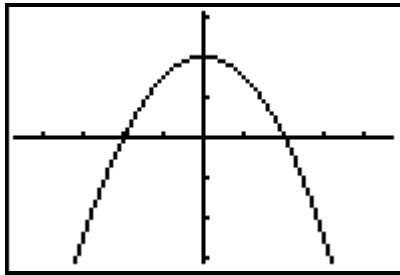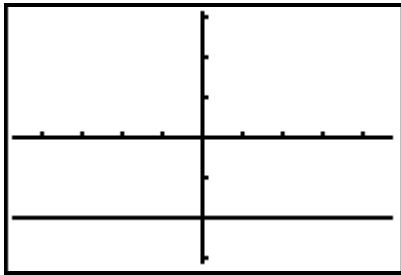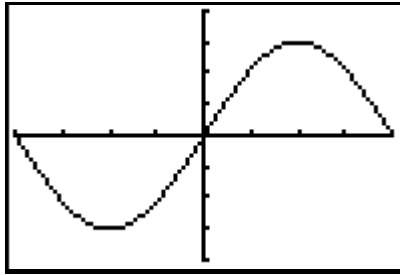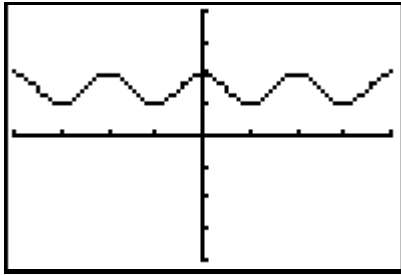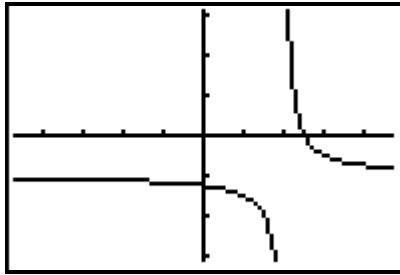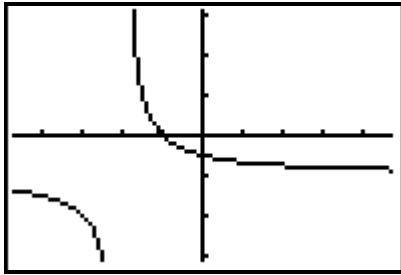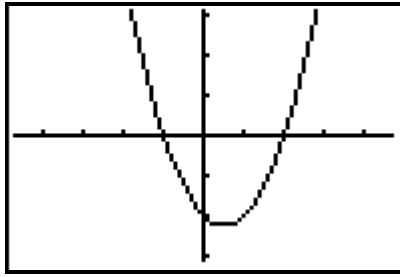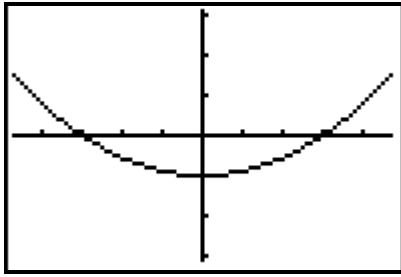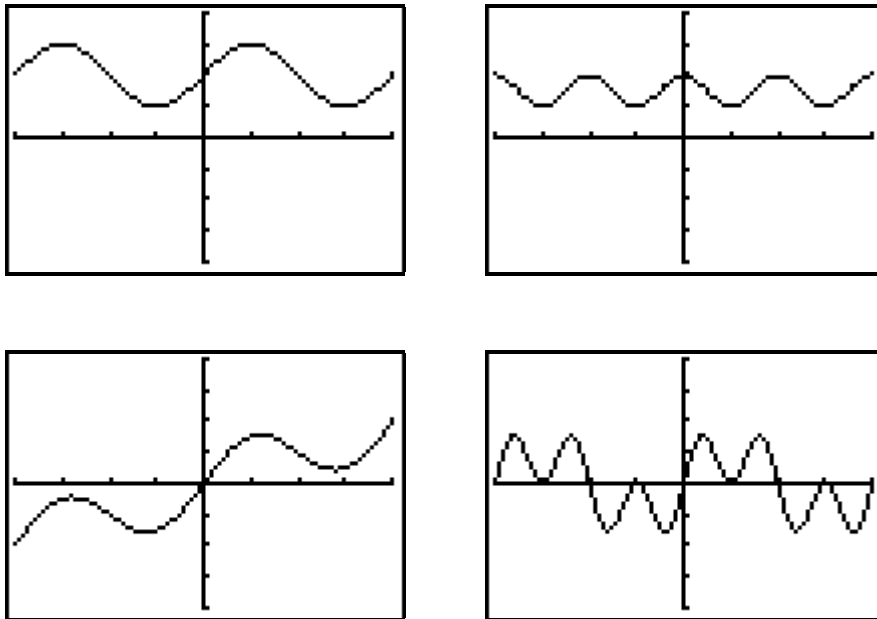
7. Parabola as locus

In studying locus we often get students to pretend to be points defined by some rule. We can think of locus as being a static thing: the set of points satisfying some condition - or as a dynamic thing: the path of a point moving to keep a condition satisfied. So, for example, you could have one student stand still to represent a fixed point O, and get students A, B, C etc. to arrange themselves so that they are each 2m from O. Alternatively you could get a student P to walk in such a way the she or he is always 2m from O. What would be the locus of a point P subtending a fixed angle, say 45°, with two fixed points A and B?

Problem 7.0 <u>Between a point and a line</u>
Here we specify a fixed line, such as a wall, and a fixed point A. We want the locus of a point P such that the distance from P to A is the same as that from P to the line. Let M be the point on the line nearest to P, so that PM is perpendicular to the line. So we want: $AP = PM$, i.e. that APM is an isosceles triangle. Hence P must lie on the perpendicular bisector of the base AM.   This now gives a construction which we can carry out in *Cabri*, or on a TI-92. If we use a line segment LL' to represent the wall, then we can choose a variable point M on LL' (i.e. M is an independent variable with LL' as domain). P is then the intersection of the perpendicular bisector of AM with the line through M perpendicular to LL'. (So P is a variable dependent on M). The locus of M with P is the required curve.
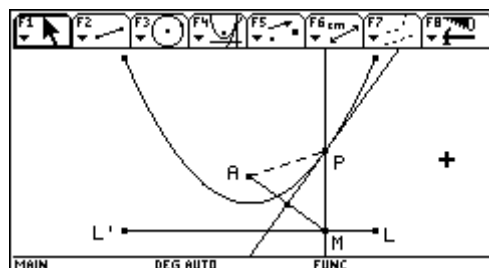
In fact the perpendicular bisector of AM is also the tangent to the curve at P, and that helps explain the reflecting property of the parabola - light coming down the line PM towards P will be reflected by the "tangential mirror" through the "focus" A - which gives the reason

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why telescopes and satelite "dish" aerials are made with parabolic cross-section. If you now choose axes such that A has co-ordinates $(0,a)$ and M has co-ordinates $(x,-a)$ can you find the corresponding co-ordinates of P, and hence identify the parabola with a polynomial function of the second degree?

Problem 7.1: <u>Some string and 2 pins</u> (other conics)
Can you derive a similar approach to constructing the locus of a point P so that the sum of its distance from two fixed points A,B is a constant $(>|AB|)$, i.e. $AP+PB = c$, and hence also finding its cartesian equation? How about the locus where it is the difference $|AP - PB|$ which is constant?

8. Solving quadratic equations by iteration
Problem 8.0: <u>Golden ratio</u>
One way of defining this is to find a number x which is one more than its reciprocal - i.e. to solve the equation $x = 1 + 1/x$. Of course you can rearrange this into the standard form of a quadratic equation and use the formula to solve it - giving two real roots, one positive one R near 1.618 and a negative one S near -0.618. A numerical approach could be to turn this into an iterative scheme: $X_{r+1} = 1 + 1/X_r$ with r = 0,1,2,3,... and some starting value X_0 .
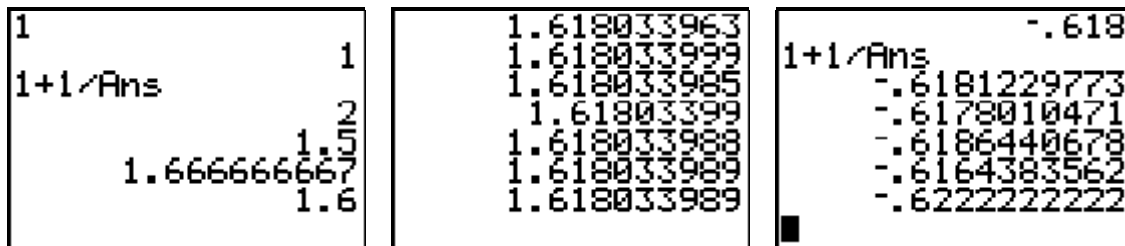
Here we see that starting with $X_0 = 1$ we converge fairly quickly to the positive root, but that even if we start very close to the negative root we get swung away, and then eventually we will home back in on the positive root. So, for this iteration it would appear that the root R near 1.618 is an "attractor" whereas the root S near -0.618 is a "repellor". Will all values of X_0 lead to sequences converging to R ? (Those which do so are called "the basin of attraction of R"). Can you find a different re-arrangement of $x^2 - x - 1 = 0$ which does produce sequences which converge to S rather than R? In order to see the situation graphically we can use the TI-83's sequence graphing mode, and produce "cobweb" and "staircase" diagrams. We need to choose a WINDOW with scales in the ratio 3:2 to ensure that the Y=X line looks like a line of gradient 1. From the FORMAT menu we select Web.
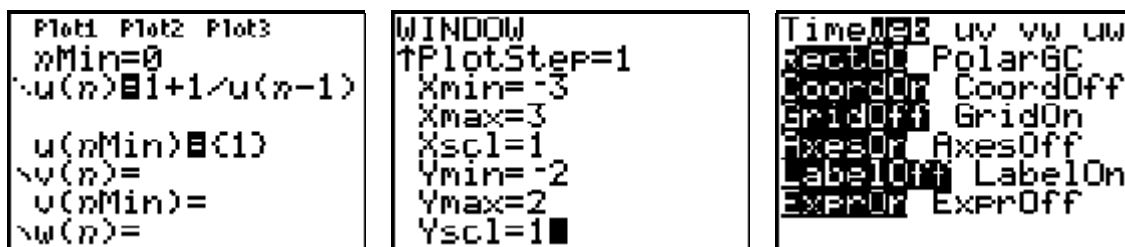
GRAPH now produces graphs both of the curve Y=1+1/X and the line Y=X. These have the two points of intersection (R,R) and (S,S) - the so-called "fixed points" of the iteration. If you press TRACE and then use the right arrow repeatedly you will see a graphic depiction of the way the iteration proceeds from a given starting value u(nMin) .
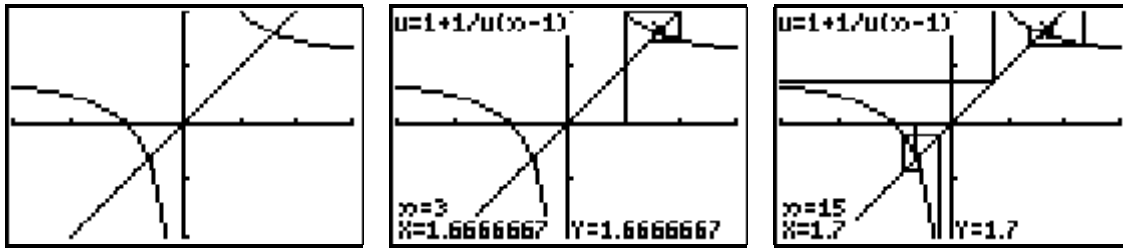
Starting with u(*n*Min) = -0.618 we see that we wind out from (S,S) until we eventually leap back to the right-hand branch of Y=1+1/X and then start to wind in towards (R,R). This is supposed to remind us of a spider's web - hence the use of the word "Web". What is it about the shape of the curve Y=1+1/X in the vicinity of (R,R) and (S,S) which determines that R is and attractor while S is a repellor? Investigate other re-arrangements of this, and other quadratics.

Problem 8.1: <u>An attracting 5-cycle!</u>
The iteration u(n) = (1+u(n-1))/u(n-2) has its fixed points given by X = (1+X)/X which yields the same quadratic equation, with roots R and S, as in Problem 8.0. This time it needs two initial values u(0) and u(1) to start. Investigate its behaviour for any choice of these starting values.

9. Approaches to the derivative
Problem 9.0: <u>Local straightness</u>
If a continuous function f(x) is differentiable at a given value of x, then, as we zoom in closer and closer around (x,f(x)) the curve will appear more and more linear, and the gradient of such a line will tend towards the value of the derivate f'(x) at that point. The screens below show successive zooms in on the graph of Y=X^2 at (1,1). The third zoom gives a good approximation to a line whose gradient we find, by tracing, to be about 2.0101415 . Can you find an approximation to the gradient at (2,4) this way?
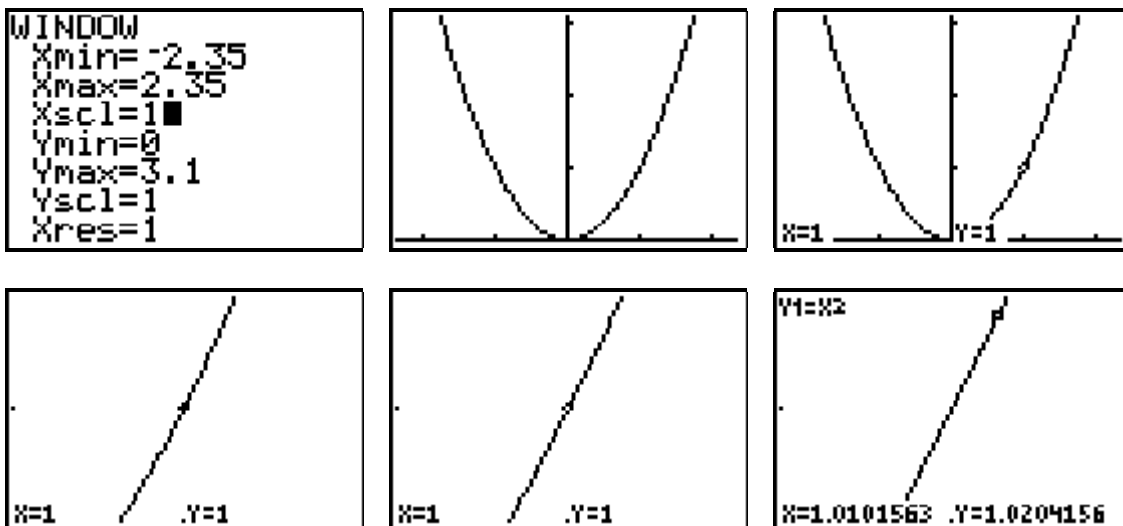


Problem 9.1: <u>The chord-slope function</u>
The chord joining the points (x, f(x)) to (x+h, f(x+h)) on the graph y = f(x) has as gradient: g(x) = (f(x+h) - f(x))/h , which, for any finite *h*, we call a chord-slope function for f. Of course one formal definition of the derivative of f at x is the limit of g as *h* tends to zero.

We need to store some value into the variable H, e.g. H=0.1 for the first graph and H=0.01 for the second. By tracing we can read of values from Y2 as approximations to the gradients at various points on the graph of Y1. We can also check that Y2 is identical to the function Y3=2X+H , which can be confirmed by algebra.
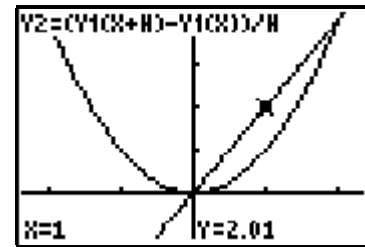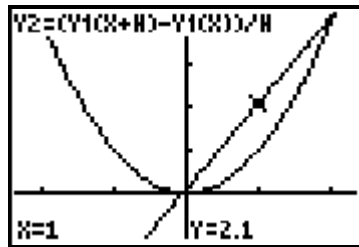
10. Verhulst and the logistic map

Problem 10.0: <u>Modelling Belgium</u>

In 1845 Pierre François Verhulst proposed a population model which he thought could predict the future population of the newly formed state of Belgium. The original model, due to a C17th English clergyman, Thomas Malthus, proposed that the rate of growth of a human population P could be modelled by what he called "geometric growth", and we call "exponential growth": dP/dt = k.P , where k>0 represents the excess of the birth-rate over the death-rate, assumed constant. Verhulst modified this to make the growth rate dependent on the population.   If the maximum population which can be sustained with available resources is Q then, as P approaches Q, Verhulst argued that the growth rate would tend to zero. So his modified model was: dP/dt = k.(1-P/Q).P

Thus when P starts small compared with Q the growth is like that of the Malthus model, but then it slows up and approaches Q asymptotically. If we write P = Qx , the equation transforms to dx/dt = k.x(1-x) , which we can convert to the numerical form:

(x(n) - x(n-1))/ Δt = k.x(n-1).(1 - x(n-1)) to turn into an iteration. Taking e.g. Δt = 1 allows us to use the TI-83 sequence mode again, this time for a TIME rather than WEB graph.
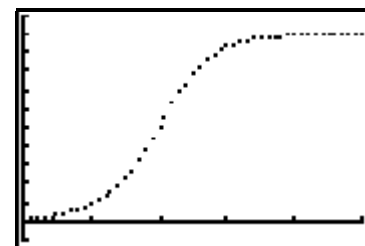

Problem 10.1: <u>chaos and the logistic map</u>

The graph above is plotted for a value of K = 0.25. But funny things begin to happen if K gets too big! See what happens e.g. for K = 2.5 or K = 3. The situation becomes literally chaotic. Yet, in essence, we are only using the simple quadratic map: f(x) = k.x.(1-x) and this is what is known as the "logistic map". Use WEB graphing on this function, arranged iteratively in the form u(n) = K*u(n-1)*(1 - u(n-1)) to investigate its fixed points, repellors and attractors for a range of values of K.

Alongside is the start of the picture for K = 3.
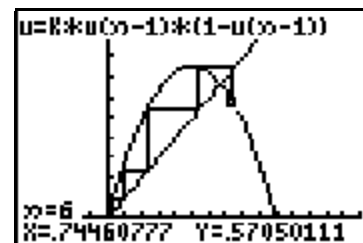
11. Solutions of differential equations

A major motivation for integration was in the solution of differential equations by anti-differentiation. The TI-85/86/89/92Plus all have tools to help visualise the numerical solution of differential equations. So a similar tool for the TI-83 might be useful.

Problem 11.0: <u>Drawing a slope field ("iron-filings") diagram</u>
In the following programs, called SLOPES, you enter the required differential equation, as a function of X and Y, in Y_1, but then deselect it! You must also specify the WINDOW you wish to use for plotting the "slope field". You can then superimpose possible solution curves in Y_2, Y_3 etc. Try using it to suggest the integral of X^2.
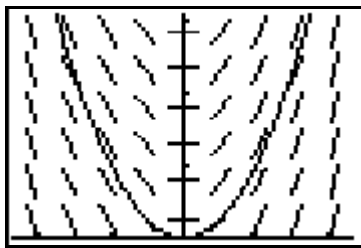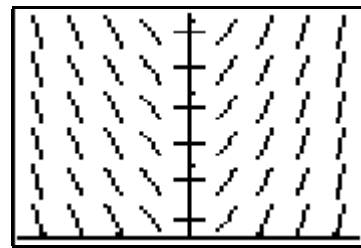
12. Numerical approaches to integration

For a function f(x) in an interval [a,b] we have a variety of ways of defining numerical approximations to the integral of f(x) in the interval based on ways of approximating the area under the graph using a finite number of strips N. Well known method are taking "left-hand" rectangles L, right-hand rectangles R, trapezia T, mid-point rectangles M and Simpson's rule S (based on quadratics). The trapezium value T is the mean of L and R. Maybe less well-known is that Simson's rule is a weighted average of T and M. In general the error using the trapezium rule T is about double that of using M, with the opposite sign. Hence taking $(2M + T)/3$ should give a much better estimate - and this is the Simpson value S. The following program, called INT, allows you to explore the various numeric estimates to the area under the function stored in Y_1 using the x-interval specified by WINDOW. As the program makes use of the Histogram plotting as a neat way of showing rectangles it can only be used with functions which are positive for all values of X in the WINDOW range. You can adapt it for more general functions, but you will have to replace the Histogram plots with loops to draw the rectangles. You will first be prompted for the number of strips.
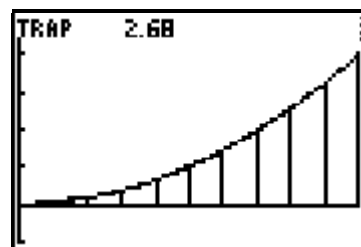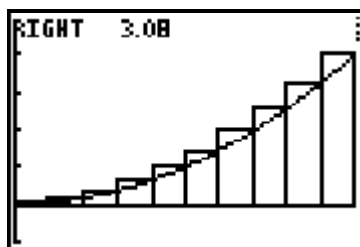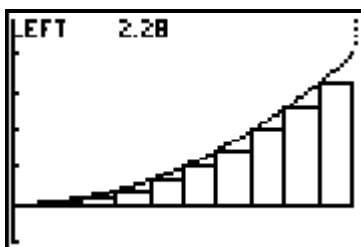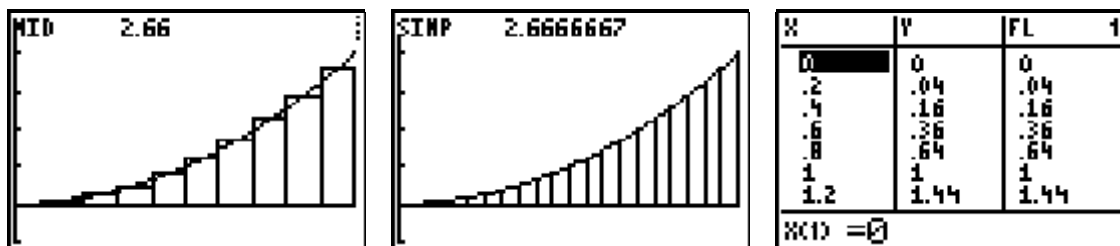
When the program finishes the lists of X and Y values, together with the "frequencies" used for the L, R, T, M and S rules, are set-up in the STAT Editor. Here is the listing of INT.

```
FnOff :PlotsOff :Prompt N: XminüA: XmaxüB: (B-A)/NüH: HüXscl
seq(P, P, A, B, H)üáX: BüáX(N+1): Y1(áX)üáY
If min(áY)<0: Then: Disp "NEG Y": Stop: End
1+Nüdim(áL): Fill(1, áL): 0üáL(N+1): áY*áLüáFL: áXüáP
Plot1(Histogram, áP, áFL): DispGraph: DrawF Y1
H*sum(áFL)üL: Text(0, 0, "LEFT"): Text(0, 30, L): Pause
áLüáR: 0üáR(1): 1üáR(N+1): áY*áRüáFR: áX-H/2üáP
Plot1(Histogram, áP, áFR): ClrDraw: DrawF Y1
H*sum(áFR)üR: Text(0, 0, "RIGHT"): Text(0, 30, R): Pause
(áL+áR)/2üáT: áY*áTüáFT: Plot1(xyLine, áX, áY, 0)
ClrDraw: For(X, A, B, H): Line(X, 0, X, Y1(X)): End
Line(B, 0, B, Y1(B)): DrawF Y1
H*sum(áFT)üT: Text(0, 0, "TRAP"): Text(0, 30, T): Pause
seq(P, P, A+H/2, B-H/2, H)üáH: B-H/2üáH(N): Y1(áH)üáFM
Plot1(Histogram, áH, áFM): ClrDraw: DrawF Y1
H*sum(áFM)üM: Text(0, 0, "MID"): Text(0, 30, M): Pause
PlotsOff : ClrDraw: DrawF Y1
For(X, A, B, H/2): Line(X, 0, X, Y1(X)): End
(2M+T)/3üS: Text(0, 0, "SIMP"): Text(0, 30, S)
SetUpEditor áX, áY, áFL, áFR, áFT, áFM
```

13. Complex maps and the Mandelbrot set

Finally we extend the idea of quadratic function to complex numbers and the complex plane. We shall just take a couple of examples inspired by Benoit Mandelbrot's quadratic iteration given by: $z_{r+1} = z_r^2 + c$, with $z_0 = 0$, for a given complex number c. So for any given value of c we can perform the iteration just using the ANS key:
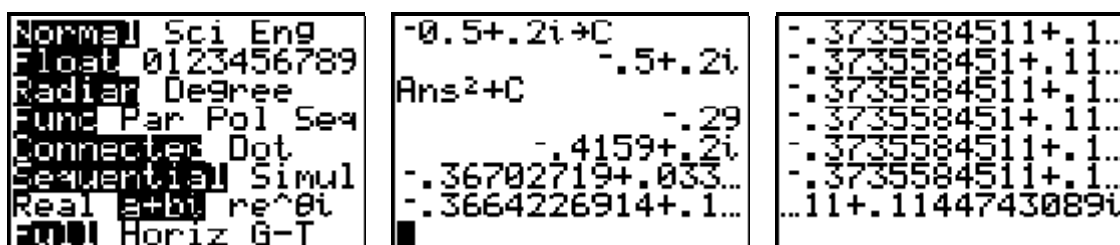
Mandlebrot sought to identify all the values of c for which this iteration did not "overflow". To interpret this geometrically he used the image of the Argand diagram for the complex plane. To illustrate a mapping from the complex plane Z to the complex plane W we would need 4 independent axes in space - a tall order. So we make do by showing what a

mapping does to some contour in the Z plane. Using parametric plotting and a suitable window we can set our object as a part of a circle of a given radius R, say. We can interpret the mapping $Z^2 + C$ geometrically using de Moivre's theorem. So a point Z gets rotated about the origin by a further angle equal to its argument, and stretched by a factor given by the square of its modulus. Then it gets translated by a vector parallel to **c**. The TI-83 display shows the mapping of the object arc under $Z^2 + C$ for the same value of C used above.
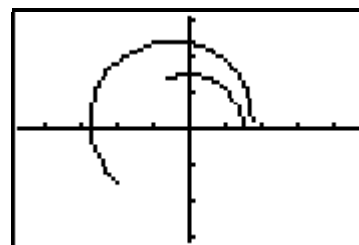

14. Postscript

I certainly didn't start out with the intention of covering all this ground. But I confess to be unsure what students in the last years of secondary school in Belgium and the Netherlands are expected to meet. I have tried to collect a variety of approaches in which the use of technology is more or less essential, and for which a graphing calculator like the TI-83 is perfectly acceptable. I have not tried at all to suggest a didactic style of use for these ideas - that is up to each individual to adapt in the way they feel happy. I hope that I have taken examples which give as much scope for thinking, experimenting, conjecturing and doing mathematics alongside the calculator as possible. I am convinced that they offer both a means of teaching conventional content better, and also in extending student's mathematics by offering greater challenges than most text-books!

Adrian Oldknow   July 28th 2000