Animated lessons with TI InterActive!

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Abstract

TI InterActive! is an ideal software to teach and learn mathematics.
Some of its characteristics as the automatic updating of documents and the existence of sliders allowing to change the values of parameters, makes TI InterActive! an excellent tool to illustrate the different situations which may occur in a given problem.
Moreover, as TI InterActive! is simple to use even for a secondary school student, worksheets can be passed on to the students, completed by them then passed on to the teacher again, all this, in the same environment. These worksheets may contain various links to other documents (partial solutions, hints, additonal informations, ...) that the students are free to consult according to their own needs. This gives them the opportunity to work in a relatively autonomous way.

In this paper, we try to illustrate the various characteristics of TI InterActive! indicated above on two subjects that have been experimented in secondary school classes (grades 9 to 12) :
The tale of the horse and the cheetah : taking as starting point the equations relative to rectilinear movements with constant velocity or with constant acceleration, the problem consists in simulating a situation in which a cheetah is pursuing a horse and in determining algebraically and graphically the issue of the pursuit.
Can you imitate the flight of a bird ?: in this work, the students are asked to simulate the flight of a bird using appropriate parametrical functions.

Important notice

Because of editing requirements, this document could not be written in a TI InterActive! environment. As a consequence, the interactivity of the TI InterActive! files in which the two above subjects were developed is lost here. This interactivity is only suggested by sequences of images when possible.

Introduction

There are several ways to use TI InterActive!:
- as a "super blackboard" to be handled by the teacher in front of the class to illustrate small lesson sequences in which the interactivity of the software provides additional value compared to normal blackboard and chalk.
- as a "dynamic" tool for the teacher to prepare his lessons at home, or for the students to perform well adapted exercises in the class or at home as described earlier in the abstract.
The tale of the horse and the cheetah fits in the first category, whereas *Can you imitate the flight of a bird ?* belongs to the second one.

The tale of the horse and the cheetah

Preliminary mathematical skills
The students should know the formulas below relative to the rectilinear motion with constant velocity (1) and to the uniformly accelerated motion (2) :

If *s* is the displacement and *v* the constant velocity

$$s(t) - s(t_0) = v.(t - t_0) \qquad (1)$$

If the initial velocity is zero, the displacement *s* and the constant acceleration *a*,

$$s(t) - s(t_0) = a.\frac{(t-t_0)^2}{2}$$ (2)

With students of the 12th grade, it is a nice exercise to "rediscover" these formulas as applications of the definite integral.

Statement of the problem
The cheetah is one of the best sprinters in the world. It is able, in ten seconds, to reach the velocity of 108 km/h and to maintain this velocity on a distance of 600 m.
But after such an effort, the cheetah gets exhausted. So, if it has not succeeded to catch the prey, it generally abandons the pursuit.
The behavior of the horse is completely different. It runs only as fast as 72 km/h but it is able to maintain this speed for many kilometers.

A cheetah doing a nap on a branch of a tree is waken up by the noise of horse's shoes passing by. When the cheetah feels fit to begin its pursuit, the horse is already 180m away from the tree. The horse is running at its optimum speed and far from beeing tired. Will the cheetah catch the horse ?

This statement is the almost litterate translation in english of an article presented in one of the editions of a popular journal for flemish teachers (see reference below). The suggestion, in this article, is to simulate the problem on a graphing calculator.
For having done it several times in my classes, I guarantee that this simulation brings a lot of animation and fun while doing mathematics.

Reference : Uitwiskeling, Jaargang 9, nummer 4, oktober 1993, « Onder de loep genomen », pages 47-49.

Solving the problem in a TI InterActive! environment
The use of TI InterActive! instead of a calculator provides an additional value I shall point out further.

First step : applying the above formulas the students
- find out easily the expression of the displacement e(t) of the horse in function of the time, cf (4) below
- find out not so easily (because it is a piecewise function) the expression of the displacement p(t) of the cheetah in function of the time, cf (5) below

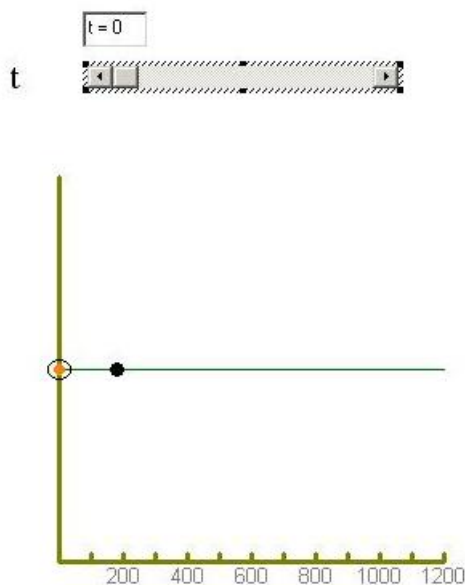Let *h* be the handicap of the horse $h := 180$ (3)
Expressing speed (resp. acceleration) in m/sec (resp. m/sec²) one has :
$e(t) := 20.t + h$ (4)

$$p(t) := \begin{cases} \dfrac{3.t^2}{2} & 0 \le t \le 10 \\ 30.t\text{-}150 & 10 \le t \le 30 \\ 750 & \text{else} \end{cases}$$ (5)
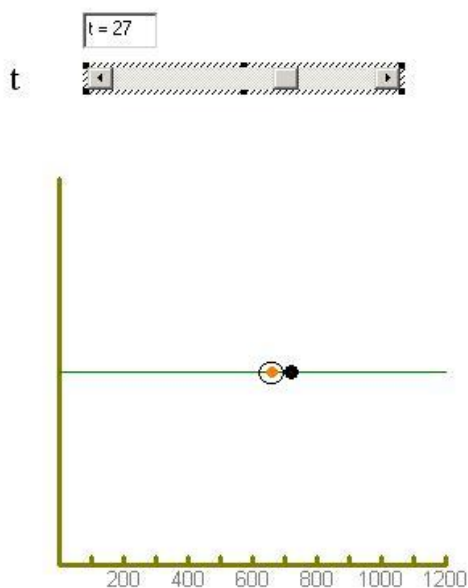
Second step : simulation of the pursuit of the horse by the cheetah.
Inserting (3), (4) and (5) in a TI InterActive! file and a *slider bar corresponding to t*, we obtain

The cheetah is represented by the small circle, the horse by the black point.
When $t = 0$, the cheetah is in the tree, the horse 180 m ahead.

When $t = 27$, the cheetah approaches dangerously the horse ...

For *t* varying between 27 and 30, moment where the cheetah stops, it is difficult to see with enough precision on the simulation if the cheetah catches the horse or not. Ones has the impression that the horse is safe, but it is not completely sure. We need to proceed in another way.

Third step : some students begin to solve equations, which is not easy for most of them again, because of the piecewisefunction. But it is interesting and worth discussing.
Others have the idea to sketch the graph of e(t) and p(t) in function of t.
But generally the graphs the students make in their notebooks are not models of precision and those who sketched the graphs are disappointed : it is still difficult to see the issue of the pursuit.

Let us make the graphs with TI InterActive!...
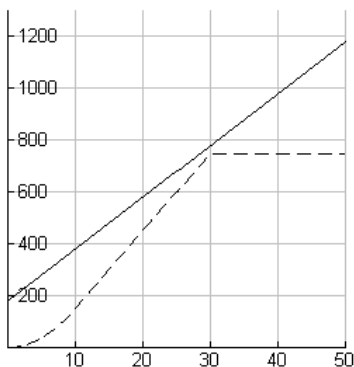
Displacement of the cheetah in dots.
Now it is clear. The cheetah does not catch the horse !

But there is always at least one student in the class who will say,
that it is a pity ... when is this poor cheetah going to eat ?

Fourth step : OK then, let us explore the problem a little further ...
Which handicaps should one give to the horse in order that the cheetah catches it ?

Here we are going to use some interesting characteristics of TI InterActive!.
If we change the value of the horse's handicap in the TI InterActive! document, *all the consequent graphs and calculations are automatically updated.*

For $h := 172$ we have
instantaneously the
following picture
which shows that
this value of h
is still too much
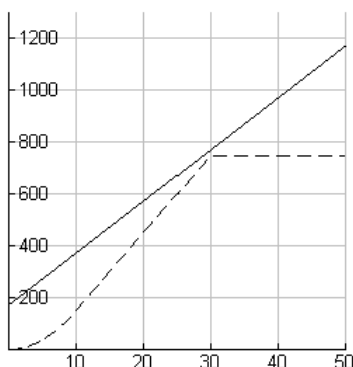
Instead of proceeding guess by guess, we should rather associate a *slider bar* to h and observe dynamically what happens.

h 149

h 8
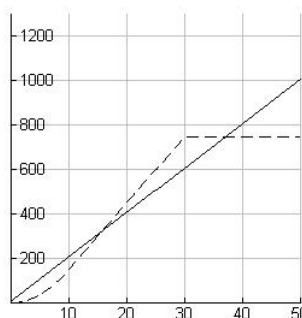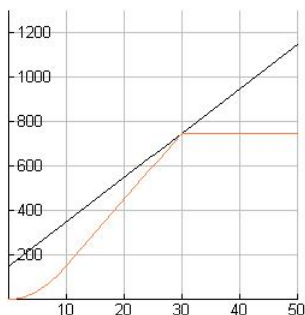
Is <u>looks</u> <u>graphically</u> as if the answer to the preceding question would be $0 \leq h \leq 149$.
What arguments should we provide to determine the largest admissible value of h ?
Clearly the question reduces to looking for the intersection of the "horse" line $20.t + h$ with the
"cheetah" line $30.t - 150$. These two lines intersect for $0 \leq h \leq 150$.

Can you imitate the flight of a bird ?

In this case, the students are supposed to work *by themselves* (eventually *by pairs*) in a TI InterActive!
environment and to produce the solution to the problem stated below in the same environment.

The material which is given by the teacher to the students consists of 4 files :
- the **main task file** contains the statement of the problem
- the **notebook file** is the file in which the students should put their solutions to the problem and write
  their explanations. This is the file the students will hand over to their teacher when the work is done.
- the **hint file** contains some mathematical suggestions that might help the students if they do not
  know how to answer question B (cf. below) of the main task file. The students are not obliged to use
  this file but are free to open it if they wish.
- the **help file** contains more technical suggestions related with syntax problems or basic
  manipulations of TI InterActive!. Again, it can be opened if the students wish so.

All these files are linked together by hyperlinks. But as the hyperlinks in TI InterActive! I are absolute
and not relative, these links will work only if the files have been put carefully in the right folder.
Another procedure is to open each of the files with a separate TI InterActive! session and to switch
from one file to another by clicking on its icon in the status bar.

<u>Preliminary mathematical skills</u>

- Knowledge of the elementary functions and their associated graphs
- Translating and stretching of curves

<u>Statement of the problem</u>

<u>Part A</u>
Use a slider associated to a parameter and use a well chosen function depending on that parameter, to
create an animated picture representing the movements of the wings of a flying bird (initial position :
wings almost horizontal, final position : wings high up) like in the figures below. Use the drawing tool
to draw the head of the bird.
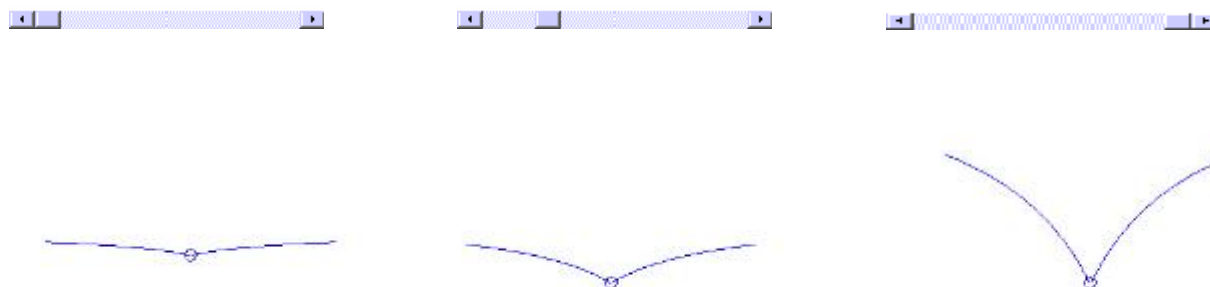
The function whose graph will represent the wings should be choosen carefully in order to avoid the effect appearing in the following example :
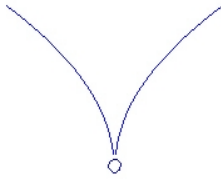
The wings seem to desolidarize themselves
from the head of the bird.
This is not good ...

Part B
We would like to elaborate a more realistic simulation of the flight of a bird : when the wings are high up, they should come down again until reaching the lowest position, then move up again, ...
Simulate some cycles of the movement of the wings of a flying bird, using <u>one slider</u> associated to a well chosen parameter.

If you need help, do not hesitate to open the hint file that has been provided.

One among many possible solutions

Part A
The head of the bird is a small circle centered at the origin.
The wings should be represented by the graph G of a function passing through the origin. In order to avoid the wings to desolidarize themselves from the head, the tangent to G should not be to close of

the y-axis. The graph of $h(x) = \dfrac{x}{x+1}$ goes through the origin and the slope of the tangent is 1 so that

this function is a good candidate.
In the image below, the slider corresponds to the parameter w varying between 0.2 and 2, the head of the bird is a circle centered at the origin and the wings are the graph of

$$f(x) = \begin{cases} w.h(x) & \text{if } 0 \le x \le 1.25 \\ w.h(-x) & \text{if } -1.25 \le x \le 0 \end{cases}$$

Part B
To simulate the movement of wings going up and down several times, the parameter *w* should be a periodic function of another parameter, let say *t*.
The slider in the following image corresponds to *t* and *t* varies from 0.5 to 20.

t = 0.95

$$w(t) := |\sin(t)| + 0.1$$

$$\text{define } f(x) = \frac{x}{x+1}$$

$$\text{define } g(x) = \begin{cases} w(t) \cdot f(x) & x \geq 0 \text{ and } x \leq 1.25 \\ w(t) \cdot f(-x) & x \geq -1.25 \text{ and } x \leq 0 \end{cases}$$

It is difficult to show it here but in a TI-Interactive environment, this bird flies !

Some realizations of students
Below you will find some of the solutions that were produced by students of the 11th or 12th grades in two schools in or near Brussels (Athénée Royal d'Uccle2, teacher Liliane Falek and Athénée Royal Riva-Bella de Braine l'Alleud, teacher Rita Levecq).