

Physics Experiments for Mathematical Education: Workshop

Hans Kammer, PhD, Physics Teacher, Berner Maturitätsschule für Erwachsene BME, CH-3012 Berne, Switzerland, hanskammer@bluewin.ch

1. Introduction

The aim of this workshop is to show to the participants how to handle some electronic sensors which can be directly (or with an EasyLink interface) connected to the Nspire system: *Motion* (distance and acceleration), *Temperature*, *Voltage* and *Force Plate*.

These sensors are automatically recognised by the software. The corresponding physical quantities are then normally measured as a function of time which can then be modelled by a mathematical function.

At the end of this workshop you should know at least one practical Nspire application with a sensor you can directly use in your class next week.

2. Discharging a Capacitor with a Resistor

An electrolytic capacitor ($C = 10'000 \mu\text{F}$) is charged with a 9-Volt-battery. Then the charging switch is opened and the capacitor is discharged with a resistor ($R = 4'700 \Omega$). The voltage across the resistor (and the capacitor) is measured with a Vernier Voltage probe which is connected to an Nspire handheld calculator with an EasyLink Interface (figures 1, 2, 3).

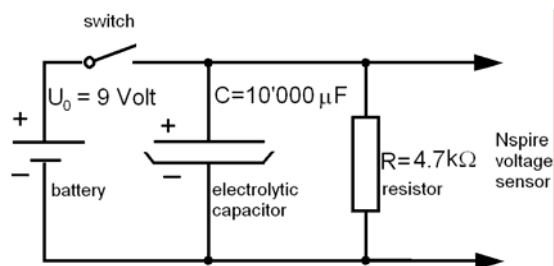


Figure 1 Discharging Circuit



Figure 2 Electrolytic Capacitor ($4'700 \mu\text{F}$), Resistor ($1'000 \Omega$) and Battery (9 Volt)

Procedure

1. Choose "New Doc" on the home page of the TI-Nspire (Vs. 2.0) handheld calculator.
2. Connect the voltage probe at one side (clips) to the R-C-circuit, on the other side to the EasyLink interface.
3. Connect the EasyLink mini-USB-connector to the TI-Nspire Handheld calculator.
4. The Calculator automatically recognises the voltage sensor.
5. Choose "Lists & Spreadsheet". A spreadsheet appears in the upper part of the screen.
6. Select the measuring line at the bottom of the calculator screen.
7. Press the menu-key, then 1: Experiment > 3: Set Up Collection > Time Graph.
8. Choose 1 second for the time between



Figure 3 Voltage Probe and EasyLink-Interface (Vernier)

A	dc01.time	B	dc01.voltage1	C	D
					=ExpReg('dc01.time','dc01.vol
1	0.	6.53931	Titel	Exponentielle Regression	
2	0.1	6.52924	RegEqn	a*b^x	
3	0.2	6.51398	a		6.55888
4	0.3	6.50391	b		0.98292
5	0.4	6.49384	r ²		0.999966
6	0.5	6.47858	r		-0.999983
7	0.6	6.47369	Resid	{-0.019571644988969,-0.018...	
8	0.7	6.46332	ResidTra...	{-0.002988453692649,-0.002...	
16	1.5000000223517				

Figure 4 Discharging and Regression Data

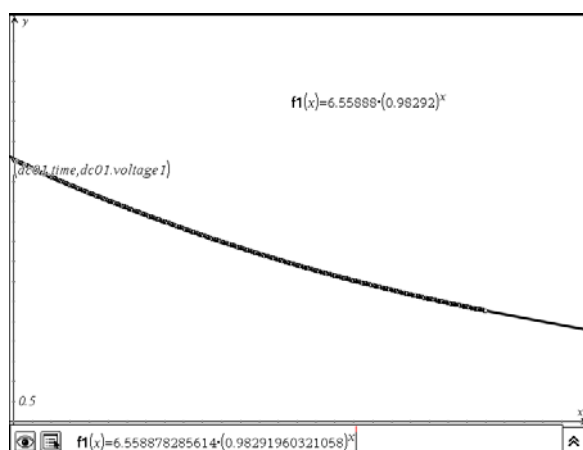


Figure 5 Voltage vs. Time: Measured Data and Regression Curve

From the exponential regression data

$$y = a \cdot b^x \quad \text{with } a \approx 6.56 \text{ Volt and } b = 0.98292 \text{ (Figures 4 and 5)}$$

the time constant τ of the discharging process can now be calculated.

Because

$$b^x = e^{-k \cdot t}$$

we get

$$\ln b = -k .$$

- the samples and 100 seconds for the Experiment Length. Press OK.
9. Charge the capacitor by shortly dipping its + pole with the + wire of the 9 volt battery. The - pole of the battery is connected with the - pole of the capacitor.
10. Press the menu-key. Select 1: Experiment> 1: Start Collection. The measurements are written in *column A* (dc01.time) and *column B* (dc01.voltage 1, figure 4).
11. Select the spreadsheet. Press the menu-key and select 4: Statistics>1: Stat Calculations>A: Exponential Regression, select dc01.time for the X List and dc01.voltage1 for the Y List. Press OK. The regression data appear in *column D* (figure 4, right).
12. On the Home-Page of the calculator select a graph page.
13. Press the menu key. Select 3: Graph Type> 4: Scatter Plot.
Enter $s1 \begin{cases} x \leftarrow \text{dc01.time} \\ y \leftarrow \text{dc01.voltage1} \end{cases}$
14. Press the menu key. Select 4: Window/Zoom > 1; Window Settings. Enter: XMin 0, XMax 100, YMin 0 YMax 10. Press OK. The measured data are indicated (figure 5).

Press the menu key. Select 3: Graph Type> 1: Function. Press enter. The regression function calculated in step 11 is now available (normally as function **f1(x)**).

For the time constant τ of this cooling process can be calculated

$$\tau = \frac{1}{k} = -\frac{1}{\ln b} = -\frac{1}{\ln 0.98202} \text{ s} = 55 \text{ seconds}$$

After 55 seconds the voltage of the discharging process reaches 37% of its initial value.

During the discharging process the voltage across the capacitor U_C and the resistor U_R at every time is the same. With the Capacitor equation $Q = C \cdot U_C$, Ohm's law $U_R = R \cdot I$ and the differential definition of electric current $I = dQ/dt$ we get the formula of the discharging process

$$U_C = U_R \text{ or } \frac{Q}{C} = -R \cdot I \text{ differentiated: } \frac{I}{C} = -R \cdot \frac{dI}{dt} \text{ solution: } I = I_0 \cdot e^{-t/R \cdot C} \text{ or } U_R = R \cdot I = \underbrace{R \cdot I_0}_{U_0} \cdot e^{-t/R \cdot C}$$

Thus the time constant is the product of the resistance R and the capacity C

$$\tau = R \cdot C = 4'700 \Omega \cdot 10'000 \mu\text{F} = 47 \text{ seconds}$$

The difference to the value obtained by the regression calculation is probably due to the inaccurate value of the capacitance of the electrolytic condenser which normally has a tolerance of about 20%.

3. Evaporation Heat of Pentane



Figure 6 Measuring the Evaporation Heat of C_5H_{12} from an absorbent Cotton Ball on a GoTemp/EasyTemp Sensor

We measure the exponential decay of the temperature measured with a temperature sensor by evaporation of pentane (C_5H_{12}) from a ball of absorbent cotton.

This decay is measured in the same way as the voltage of a capacitor during its electrical discharging.

As temperature sensor we use a GoTemp for a computer and an EasyTemp for a calculator.

The temperature and time values again are measured with the aid of a spreadsheet and are displayed in a scatter plot (as in figures 4 and 5 for the electrical voltage).

In this experiment the temperature falls from an initial value of $\vartheta_{\text{initial}} \approx +20^\circ\text{C}$ to final value of $\vartheta_{\text{environ}} = -17.2^\circ\text{C}$ about. This temperature curve can be described by

$$\vartheta(t) = \vartheta_{\text{environ}} - (\vartheta_{\text{environ}} - \vartheta_{\text{initial}}) \cdot e^{-k \cdot t}$$

The temperature decay curve this time will be analyzed by “optical fitting” this time: We start with a spreadsheet and a scatter plot of the temperature data (as in figures 4 and 5):

Procedure

1. In the graph (scatter plot) representation of the data press the menu-key and select 3: Graph Type >1: Function.
2. Insert $f_1(x) = t_e - (t_e - t_i) \cdot e^{-k \cdot x}$.
3. Press the menu-key and select 4: Window/Zoom > 1: Windows Settings Xmin 0, Xmax 250, Ymin -20, Ymax 30. OK.
4. Press the menu-key and select 1: Actions >A: Insert Slider.
5. Select the slider. Press the ctrl menu key. Select 1: Settings... . Enter Variable t_e , Value -15, Minimum -30, Maximum 0, Step Size 1, Style Horizontal, OK.
6. Do the same for the variables t_i (Value 20, Minimum 0, Maximum 40, Step Size 1) and k (Value 0.01, Minimum 0, Maximum 0.03, Step Size 0.001).

With the sliders you now can fit the graph of $f_1(x)$ with the scatter plot of the measured temperatures and read out the slider values.

These results can be compared with those of a exponential regression analysis.

4. Pendulum



Figure 7 Distance Sensor



Figure 8 1-d Accelerometer (Vernier). The Arrow on the Sensor shows the Measurement Direction.

The oscillations of a pendulum can be measured with two methods.

1. Contactless with a CBR-2- (Texas Instruments) or a GoMotion- (Vernier) ultrasonic distance sensor (figure 7) or
2. with a 1-d- accelerometer which is glued to the pendulum bob by double sided scotch tape.

The measurement and data analysis procedure is the same as in section 2 (capacitor discharge) and section 3 (evaporation heat of Pentane). For distance measurements with the CBR-2 sensor and accelerometer data sinusoidal regression and “optical fitting” are possible.

In the evaluating program for the distance sensor velocity and acceleration data are calculated by finite differences of the measured distances. As a consequence of this calculation method the velocity and particularly the acceleration data are much noisier than the original distance data.

This disadvantage can be avoided by measuring with an acceleration sensor. This probe has to be fixed very carefully however, because the pendulums oscillations could be influenced by the sensor cable.

5. R-L-C-Oscillator (Oscillating Circuit)

A slowly oscillating electric circuit can be realised with a 500 H-/630 H-high inductivity coil (Leybold 517 011), a 40 μF capacitor (Leybold 517 021) and a 9-Volt-battery (figures 9 and 10).

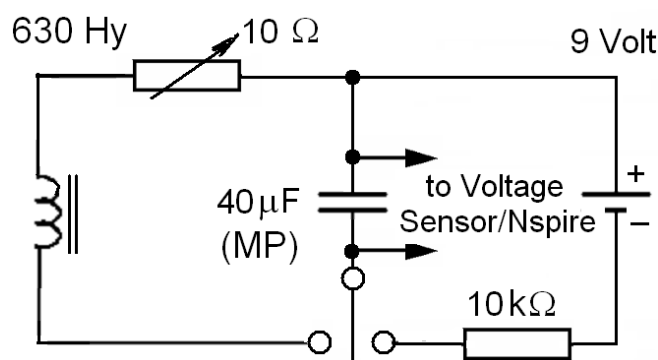


Figure 9 Slowly oscillating Circuit (1 Hertz)

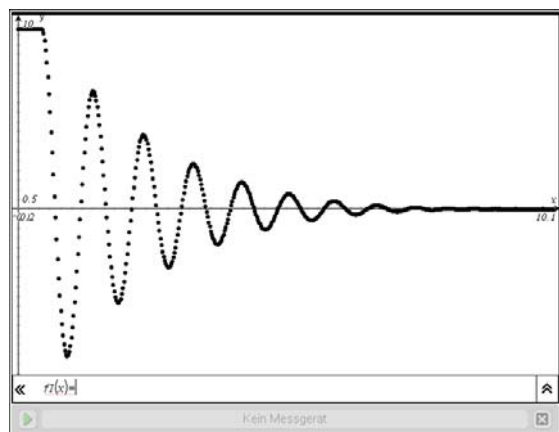


Figure11 Damped Electric Oscillation

Following Kirchhoff's Voltage rule the sum of the three voltages across the coil, the capacitor and the resistor must be zero:

$$U_R + U_C + U_L = 0 \rightarrow \underbrace{R \cdot I}_{\text{Ohm}} + \underbrace{\frac{Q}{C}}_{\text{Capacitance C}} + \underbrace{L \cdot \frac{dI}{dt}}_{\text{Self Inductance}} = 0 \quad \text{differentiated:} \quad R \cdot \frac{dI}{dt} + \frac{I}{C} + L \cdot \frac{d^2I}{dt^2} = 0$$

or

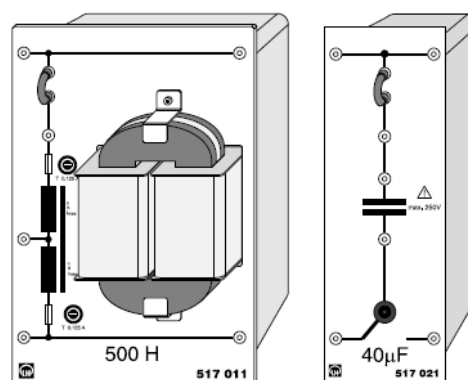


Figure 10 High Inductivity Coil (500 H) and Capacitor (Leybold 517 011/021)

The resulting damped oscillations (figure 11) can be described by the function

$$U(t) = U_0 \cdot e^{-k \cdot t} \cdot \cos(\omega \cdot t)$$

for the voltage $U(t)$. Again this function can be evaluated by "optical fitting" as in the examples in sections 3 and 4.

Nspire has no data model for the regression of this function.

$$\frac{d^2 I}{dt^2} + \frac{R}{L} \cdot \frac{dI}{dt} + \frac{1}{L \cdot C} \cdot I = 0$$

This is the differential equation of a damped harmonic oscillator with the solution

$$I(t) = I_0 \cdot e^{-k \cdot t} \cdot \cos(\omega \cdot t) \text{ and } U_R = \underbrace{I_0 \cdot R}_{U_0} \cdot e^{-k \cdot t} \cdot \cos(\omega \cdot t) \text{ with } k = \frac{R}{2 \cdot L} \text{ and } \omega = \frac{1}{\sqrt{L \cdot C}} \text{ (Thomson)}$$

The voltage functions $U_C(t)$ across the resistor R and $U_L(t)$ across the inductance L can be calculated by integration or differentiation respectively. The k – and the ω – values remain the same.

Analysing the damped oscillation (figure 11) the k – and the ω – values can be determined by “optical fitting”. If one of the three electrical portions R , L or C is known, the others can be calculated.

6. Faraday’s Induction Law

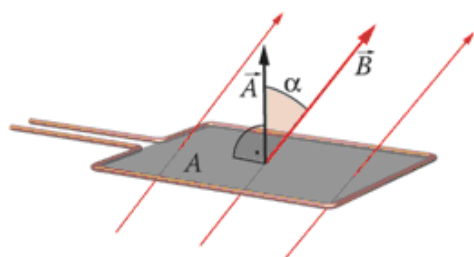


Figure 12 Faraday’s Induction Law

The induced voltage U_{ind} in a closed circuit is equal to the time rate of change of the magnetic flux Φ_m through the circuit (figure 12)

$$U_{ind} = - \frac{d\Phi_m}{dt} \quad \text{with}$$

$$\Phi_m = \vec{B} \cdot \vec{A} = A \cdot B \cdot \cos \alpha$$

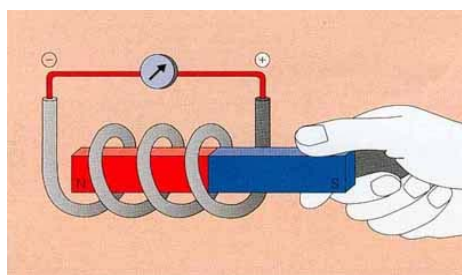


Figure 13 Moving a Magnet through a Coil

If a magnet is moved through a coil (figure 13) a voltage is induced which depends on the *velocity* of the magnet.

If the magnet is not moved no voltage is induced.

The inductive voltage generated in a coil can be measured with the voltage sensor, the EasyLink adapter and a Nspire handheld calculator.

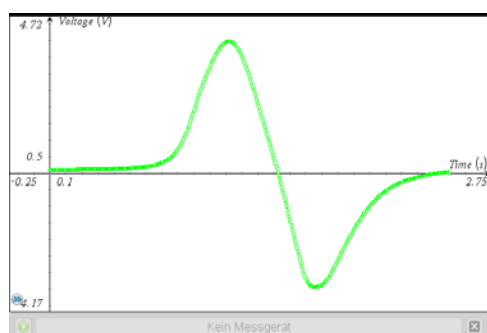


Figure 14 Uniform Motion of a Magnet through a Coil: Electrical Signal (Voltage)

If a ripple (50 Hertz) disturbing signal should be observed a R-C-low-pass filter can be used ($R = 47\text{k}\Omega$, $C = 2.2\mu\text{F}$).

Figure 14 shows the electrical signal of a magnet uniformly moved through a coil with 23'000 turns (Leybold 562 17, figure 15). How can this dispersion-like signal be modelled?

Figure 17 shows an oscillator experiment with a bar magnet, figure 16 the corresponding signal showing two oscillating modes, one longitudinal the other transversal. How could we model this experiment by an appropriate mathematical function?



Figure 15 Coil (23'000 turns), Bar Magnet, R-C-Filter and EasyLink Adapter

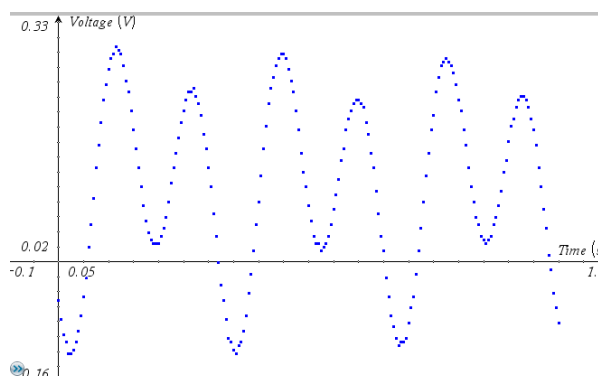


Figure 16 Twin Mode Signal of an Oscillating Magnet in a Coil: How to model?

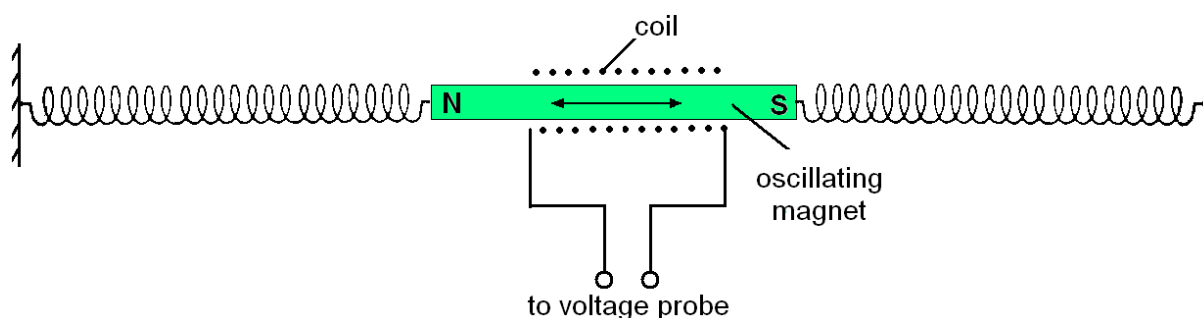


Figure 17 Oscillating Magnet in a Coil

7. Electric Characteristics (Diode, Bulb)

To measure the characteristics of an electronic/electric component, e.g. a resistor, a diode (LED) or a filament bulb, *two* measurements have to be done, one for the voltage, the other for the electric current. At the moment only one measurement at a time can be done with the Nspire system. Texas Instruments has announced for 2011 an interface system for Nspire which will allow multiple measurements.

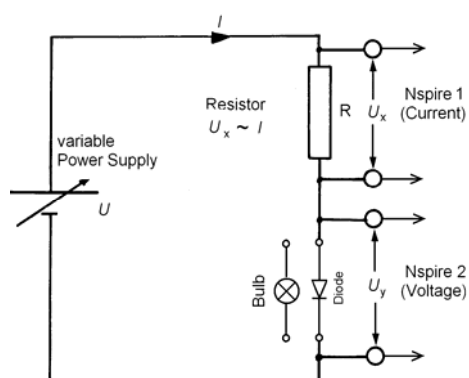


Figure 17 Circuit to measure Electric Characteristics

Several calculators have to be used therefore in order to perform multiple measurements with the actual Nspire system.

For example, one to measure the current, the other to measure the voltage (figure 17).

The measurements have then to be gathered and analysed on a single calculator.

Figure 18 shows the characteristics of a red LED with the typical “knee-voltage” (anti-diffusion voltage) of the GaAs/GaP

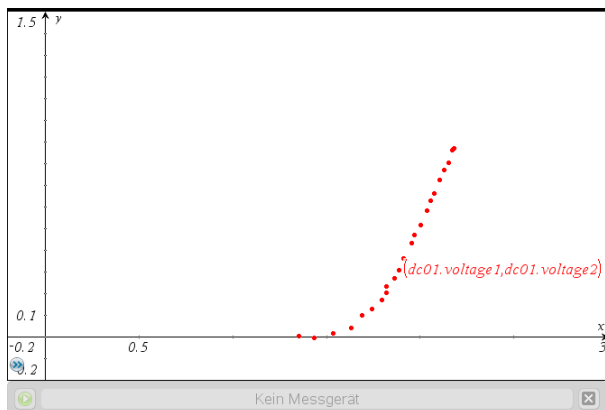


Figure 18 Characteristics of a red LED

semiconducting crystal at about 2 volt. This graph can be modelled by an exponential function.

A bulb has a positive temperature coefficient, this means that the resistance at room temperature is much smaller than at its working temperature.

The corresponding characteristics is a power function as can be shown with two assumptions:

1. The resistance R of the bulb is proportional to the absolute temperature T of the filament $R = \frac{U}{I} = c_1 \cdot T$.
2. Following Stefan Boltzmann's law the (radiation) power is proportional to the 4th power of the absolute temperature

$$P = U \cdot I = \sigma \cdot S \cdot T^4 \quad (S \text{ Surface of the bulb})$$

$$\rightarrow U \cdot I = \sigma \cdot S \cdot \frac{U^4}{c_1^4 \cdot I^4}$$

$$\rightarrow I^5 \propto U^3 \quad \text{or} \quad I \propto U^{0.6}$$

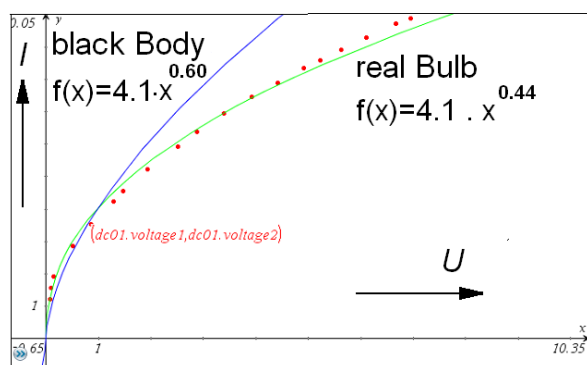


Figure 19 Characteristics of a Filament Bulb

Figure 19 shows the ideal (blue) and real (green) characteristics of a bulb. The real bulb values follow fairly well a power function, but its exponent has only a value of 0.44 (gray body) instead of 0.60 (black body).

8. Force Platform

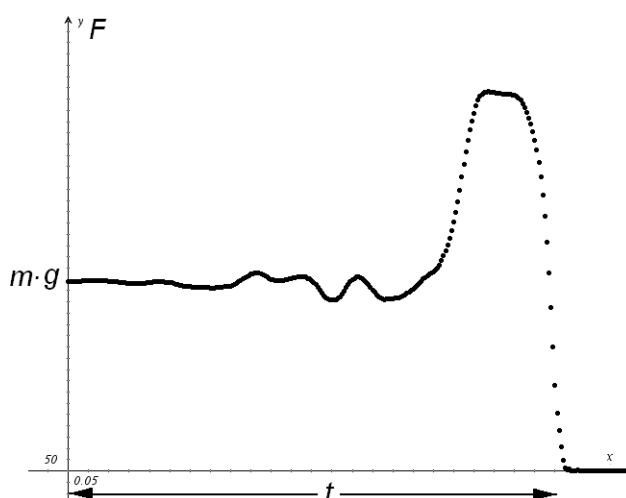


Figure 20 Jumping from the Force Plate: Force to Time Diagram

Analysing the diagram of a jump from a force platform, the height h of this jump (center of mass) can be calculated with the linear momentum Δp and the conservation of mechanical energy:

Linear Momentum (Impulse):

$$\Delta p = m \cdot \Delta v = \int_0^t (F - m \cdot g) \cdot dt$$

Conservation of Mechanical Energy:

$$\frac{1}{2} \cdot m \cdot \Delta v^2 = m \cdot g \cdot h$$

Height of the Jump :

$$h = \frac{\Delta v^2}{2 \cdot g} = \frac{\left(\int_0^t \left(\frac{F}{m} - g \right) \cdot dt \right)^2}{2 \cdot g}$$

Figure 20 shows some measured force values, figure 21 the numerical calculations with Nspire. The numerical calculation of the integral has been performed by the trapezoidal rule (figure 22).

	A dc01.time	B dc01.force1
1	0.	839.691
2	0.005	839.691
3	0.01	838.47
4	0.015	839.691
5	0.02	839.691
6	0.025	839.691
7	0.03	842.285
A7	0.	

Figure 21 Measured Force to Time Values Pairs

$f(n) = \sum_{i=2}^n \left(\frac{dc01.force1[i+1] + dc01.force1[i]}{2} \cdot (dc01.time[i] - dc01.time[i-1]) \right)$	
$h(n,m) = \frac{\left(\frac{f(n)}{m} - 9.81 \cdot dc01.time[n] \right)^2}{2 \cdot 9.81}$	
$h(249,84)$	0.138419
©Sprunghöhe 13.8 cm	

Figure 22 Evaluation of a Jump from the Force Plate

9. Conclusions

In this workshop we analyzed real world data by fitting them with appropriate mathematical functions. We worked with “optical fitting” and regression. If regression is done with students the ideas of *least square fitting* (C. F. Gauss) should be discussed.

We worked with mechanical, thermo-dynamical and electro-magnetic examples:

Pendulum, Force Plate, Evaporation Heat, Discharging a Capacitor, Oscillating Circuit, Characteristics of electric/electronic Component and Induction Law.

When working with students it’s not absolutely necessary that they know the physical background. But the instructor should know it.

The new calculator technology with the possibility to measure many types of real world data directly in the classroom allows to teach mathematics on a much broader, practical foundation. This is a real challenge – for students and for teachers!