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Workshop „Difference Equations"

Part 2:

Long-term behaviour of linear difference equations

Definition: $y_{n+1} = a \cdot y_n + b$ are called linear difference equations

Mathematical models of the following growth processes are linear difference equations:

	Linear difference equation $y_{n+1} = a \cdot y_n + b$	
Exponential growth	$y_{n+1} = q \cdot y_n$	a = q and b = 0
Limited growth	$y_{n+1} = y_n + r \cdot (G - y_n)$ $y_{n+1} = (1-r) \cdot y_n + r \cdot G$	a = (1-r) and b = r·G
Growth with intervention	$y_{n+1} = y_n + r \cdot y_n - e$ $y_{n+1} = (1+r) \cdot y_n - e$	a = (1+r) and b = e

Calculate the fixed point y* of the difference equation (y* = f(y*)) and investigate the long-term behavior of the difference equation for following cases (y_0:is the initial value):

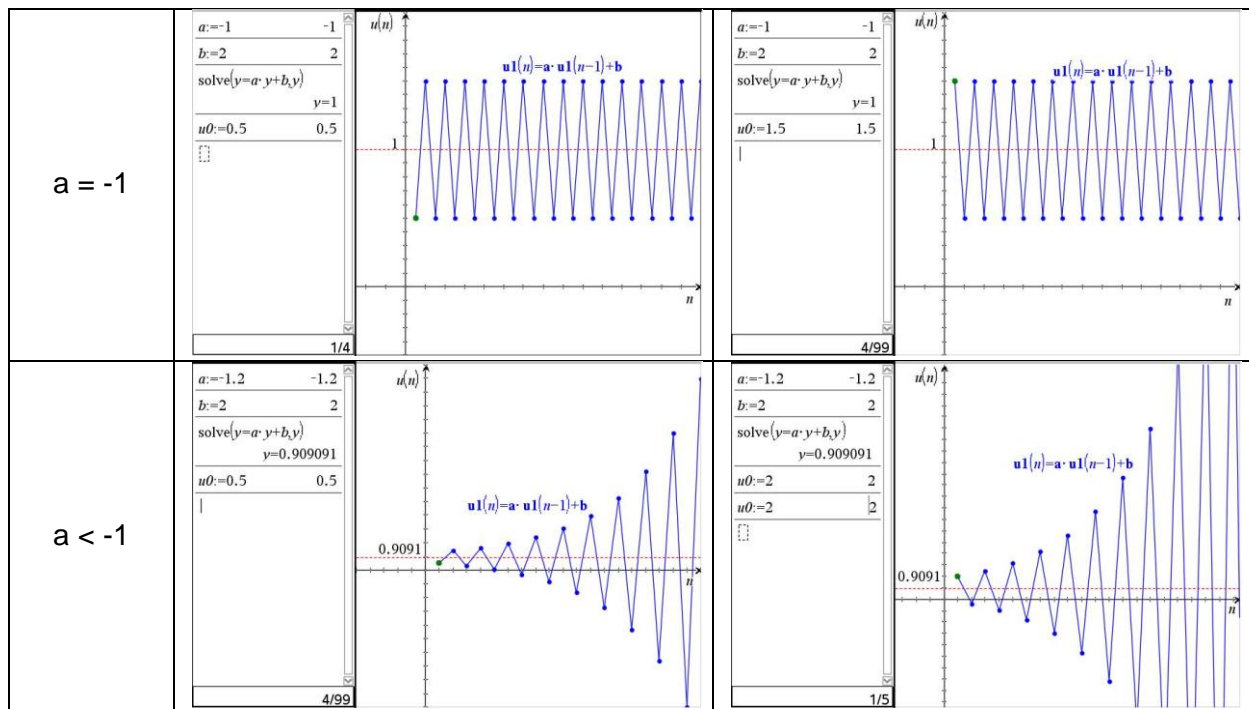|	$y_0 < y^*$	$y_0 > y^*$
a > 1		
0 < a < 1		
a = 0		
-1 < a < 0		
a = -1		
a < -1		

Select the initial point in the respective interval and draw the suitable graphs in the "time mode" and the "web mode". A possible value for b is 2, but for a>1 it is better to take b=-2.

Choose the suitable window variable; in the "web mode" it is necessary to choose "zoom square" to get the correct position of the 1st median.

Expected solutions:

a) Graphs in the "time mode"

	$y_0 < y^*$	$y_0 > y^*$
a > 1		
0 < a < 1		
a = 0		
-1 < a < 0		

a = -1		
a < -1		

b) Graphs in the "web mode"

	$y_0 < y^*$	$y_0 > y^*$
a > 1		
0 < a < 1		

a = 0	$a:=0$ 0
	$b:=2$ 2
	$solve(y=a\cdot y+b,y)$
	      $y=2$
	$u0:=1$ 1
	
	4/99
		$a:=0$ 0
		$b:=2$ 2
		$solve(y=a\cdot y+b,y)$
		      $y=2$
		$u0:=4$ 4
		
		4/99

-1 < a < 0	$a:=-0.9$ -0.9
	$b:=2$ 2
	$solve(y=a\cdot y+b,y)$
	      $y=1.05263$
	$u0:=0.5$ 0.5
	$u0:=0.5$ 0.5
	$u0:=0.5$ 0.5
	#
	6/99
		$a:=-0.9$ -0.9
		$b:=2$ 2
		$solve(y=a\cdot y+b,y)$
		      $y=1.05263$
		$u0:=1.5$ 1.5
		|
		
		4/99

a = -1	$a:=-1$ -1
	$b:=2$ 2
	$solve(y=a\cdot y+b,y)$
	      $y=1$
	$u0:=0.5$ 0.5
	|
	
	4/99
		$a:=-1$ -1
		$b:=2$ 2
		$solve(y=a\cdot y+b,y)$
		      $y=1$
		$u0:=1.5$ 1.5
		|
		
		4/99

a < -1	$a:=-1.2$ -1.2
	$b:=2$ 2
	$solve(y=a\cdot y+b,y)$
	      $y=0.909091$
	$u0:=2$ 2
	|
	
	4/99
		$a:=-1.2$ -1.2
		$b:=2$ 2
		$solve(y=a\cdot y+b,y)$
		      $y=0.909091$
		$u0:=2$ 2
		|
		
		4/99