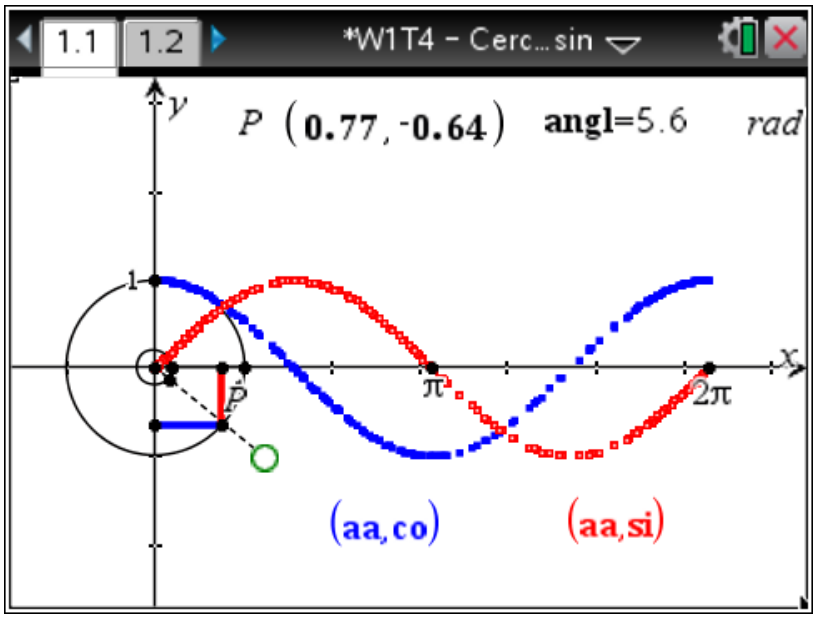
T³ VLAANDEREN

18de T3 Vlaanderen Symposium Oostende

24 & 25 augustus 2015

Functies introduceren met CAS in wiskundelessen in de tweede graad

Luc Blomme en Alain Jaeckel



TEXAS INSTRUMENTS

KATHOLIEKE UNIVERSITEIT LEUVEN

Functies introduceren met CAS in wiskundelessen in de tweede graad

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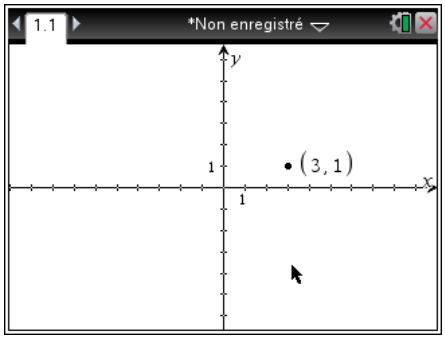
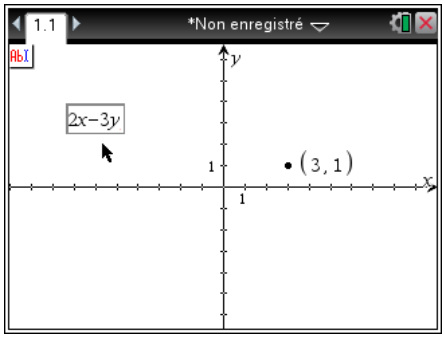
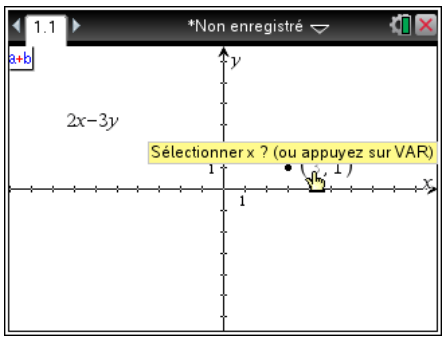
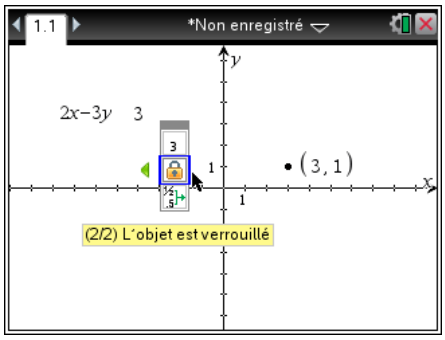
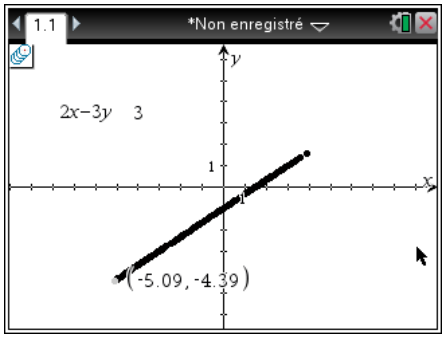
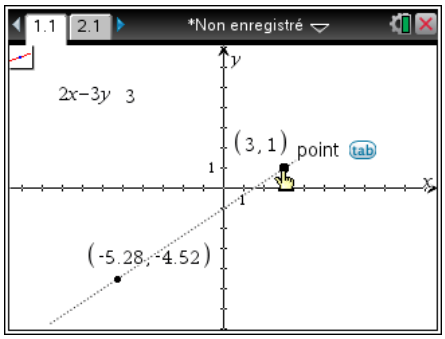
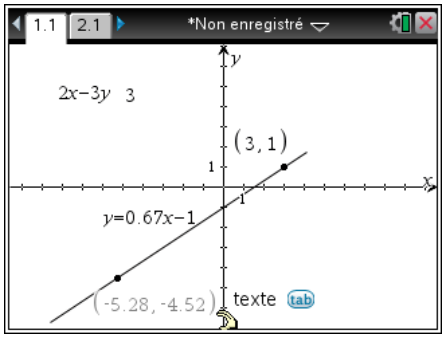
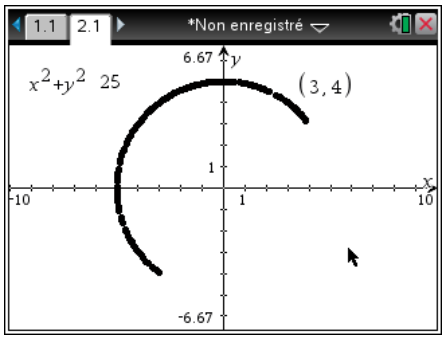
Activity 1: Link the points whose coordinates lie on a straight line given its equation

Learning objectives: lines, gradient, equations, plotting sine functions using scatter plots

Open a "Graphs" window. Plot the coordinate (3 ; 1): "menu 8: Geometry – 1: Points and lines – 1: Point – (3 enter 1 enter". To display the coordinates move the cursor over the point then "ctrl menu 7: Coordinates and Equations".	
Insert a text box: "menu 1: Actions – 7: Text". Click a location on the page and write "2*x* – 3*y*".	
Set the calculator to interpret the text as an algebraic expression that can be calculated given the *x* and *y* values: "menu 1: Actions – 9: Calculate". Move the cursor over the text "2*x*-3*y*" and click. Use the coordinate (3,1) to specify the *x* and *y* values of the expression. Place the result (3) next to the text. Show the students that when the original point is moved, the value of the calculation changes. Move the coordinate back to (3,1).	

Now we are going to keep the value of the expression fixed at 3. Place the cursor over the value 3 then "ctrl menu 2: Attributes" and select object is locked.	
Move the point and observe what happens. Then place a trace on the point: "ctrl menu 9: Geometry trace" (click to select the point).	
Possible extensions Draw a line by selecting two points of the line: "menu 8: Geometry − 1: Points and lines − 4: Lines".	
To obtain its equation, move the cursor over the line and "menu 7: Coordinates and equations". Show that the two equations are equivalent.	
Another example: $x^2 + y^2 = 25$.	

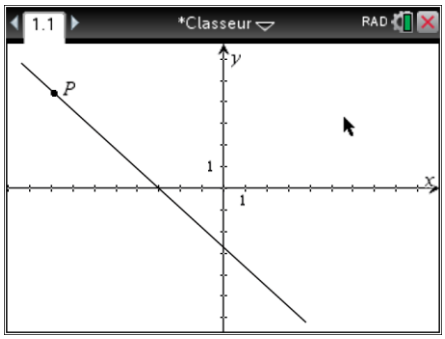
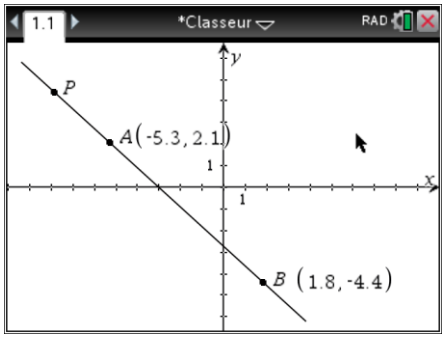
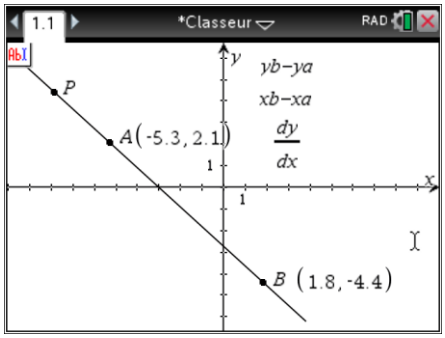
Activity 2: Game "Place the points in the correct position"

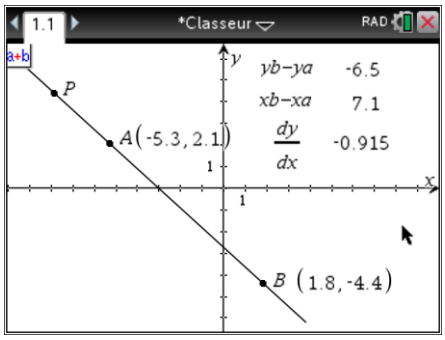
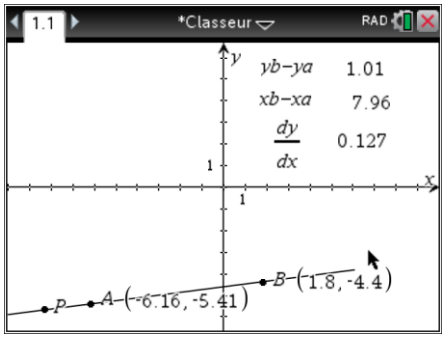
Open the *tns* file "*W2 - Line traffic lights*".

This activity allows the student to find coordinates which satisfy the line equation $ax + by = c$ where *a* ,*b* and *c* are integers. Feedback is given to the student for each coordinate chosen using a traffic light method: a green light for correct, amber for close and red for too distant. The values of *a* ,*b* and *c* can be changed and the game can be repeated.

Possible extension: $(x - a)^2 + (y - b)^2 = c$.

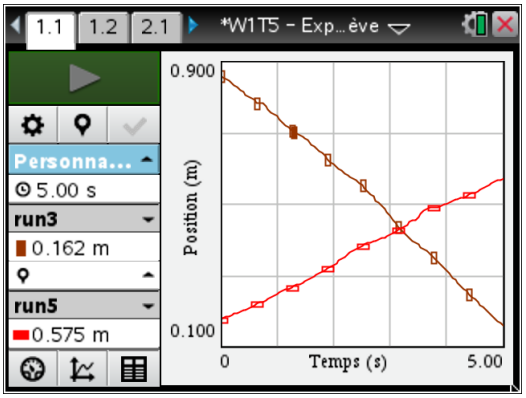
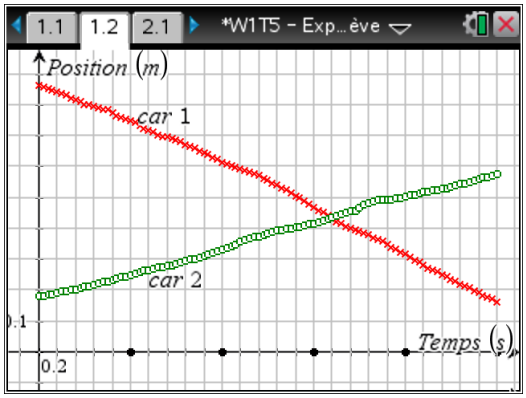
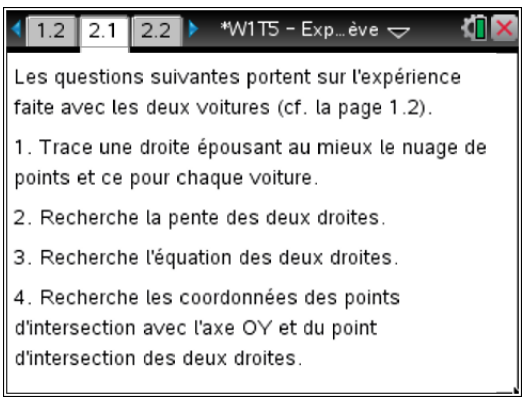
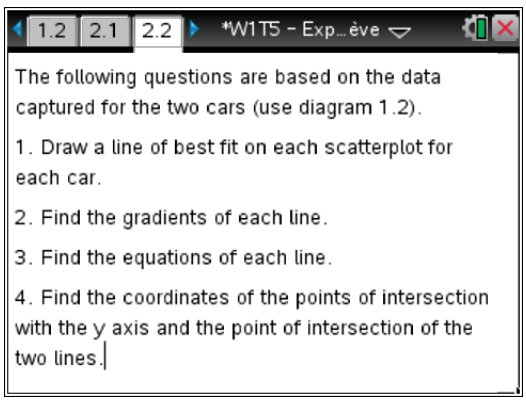
Activity 3: Gradient of a line

Open a graphics window Place a Point P and a line passing through P: "menu 8: Geometry – 1: Points and lines – 1: Point" then: "menu 8: Geometry – 1: Points and lines – 4: line". Observe how to move the line.	
Put two points A and B on the line and obtain their coordinates: "menu 8: Geometry – 1: Points and lines – 2: Point on". It is wise to label the points as you go. To obtain the coordinates, move the cursor over the point then "ctrl menu 7: Coordinates and equations".	
We are now going to calculate the difference in the y coordinate of the two points A and B, then the difference in the x coordinated and then the quotient of the two differences. Insert text boxes: "menu 1: Actions – 7: Text". Use the three different text boxes "*yb – ya*", "*xb – xa*" et "$\frac{dy}{dx}$".	

Display the results of each calculation by “Calculate”: “menu 1: Actions – 9: Calculate”. Using the pointer, show le calculation then the difference variables required one by one.	
Change the points *A, B, P* or the line and notice what happens. You can also find the equation of the line or the gradient of the line.	

<u>Activity 4: Data capture and test using the TI-Navigator</u>

Open the *tns* file “*W4 – Experience auto test student*” and answer the questions.

2.2 2.3 2.4 *W1T5 - Exp...ève

FR – Quelle est la pente de la droite représentant la position de la voiture 1 ?
EN – What is the gradient of the line representing the position of car 1?

Entrer un nombre

2.2 2.3 2.4 *W1T5 - Exp...ève

FR – Quelle est la pente de la droite représentant la position de la voiture 2 ?
EN – What is the gradient of the line representing the position of car 2?

Entrer un nombre

2.3 2.4 2.5 *W1T5 - Exp...ève

FR – Quelle est l'équation de la droite représentant la position de la voiture 1? Choisir la réponse la plus proche de la vôtre.
EN – What is the equation of the line representing the position of car 1? Choose the closest answer from the list below.

- y=−0.14x+0.86
- y=14x+0.88
- y=x+2

2.4 2.5 2.6 *W1T5 - Exp...ève

FR – Quelle est l'équation de la droite représentant la position de la voiture 2 ? Choisir la réponse la plus proche de la vôtre.
EN – What is the equation of the line representing the position of car 2? Choose the closest answer from the list below.

- y=x−6
- y=−0.9x
- y=0.09x+0.2

2.5 2.6 2.7 *W1T5 - Exp...ève

FR – Les deux voitures ont-elles la même vitesse ?
EN – Are the two cars travelling at the same speed?

- Oui / Yes
- Non / No

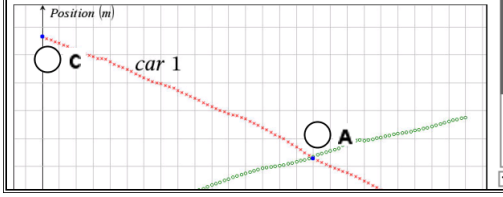
2.6 2.7 2.8 *W1T5 - Exp...ève

FR – Quelle est l'ordonnée du point d'intersection des deux droites ?
EN – What is the y coordinate of the point of intersection of the two lines?

Entrer un nombre

2.7 2.8 2.9 *W1T5 - Exp...ève

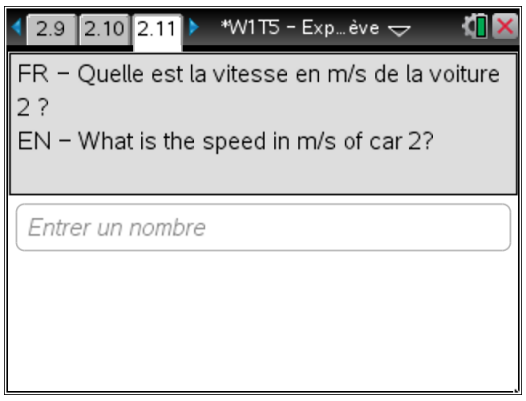
FR – A l'aide de quel point peut-on trouver la position de la voiture 1 au démarrage du chrono ?
EN – Which point gives the position of car 1 at the start of the journey?

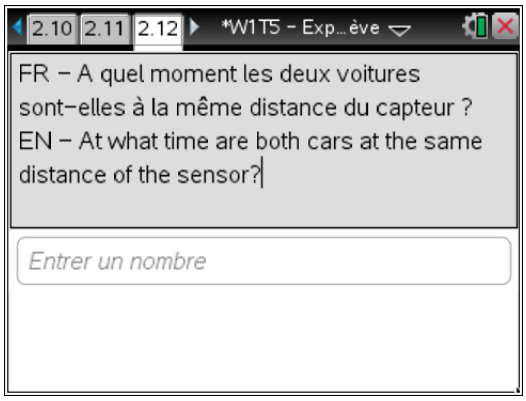


2.8 2.9 2.10 *W1T5 - Exp...ève

FR – Quelle est la vitesse en m/s de la voiture 1 ?
EN – What is the speed in m/s of car 1?

Entrer un nombre





Activity 5: Angry birds and second degree regression

To insert an image on the graphics background chose an image in the menu and then you can insert an image file.

For this activity you can also open the Open the *tns* file "*W5 – Angry birds"*. This graphic is the background from the game angry bird which shows the trajectory of a bird thrown from a catapult to hit a pig target. This activity will study the trajectory of the bird missile which is a parabola.

Learning activities – questions to consider

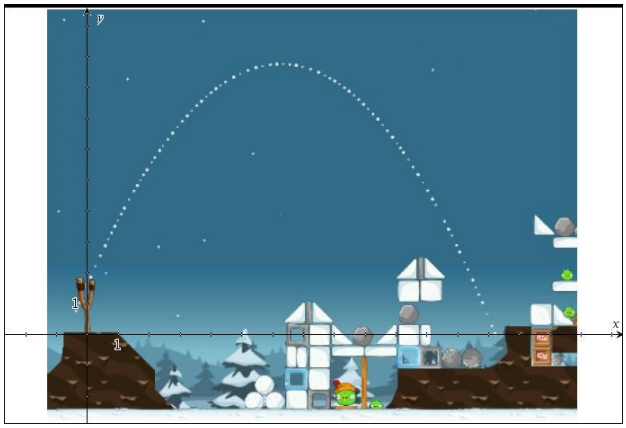
- What height is the angry bird catapulted from?
- What is the maximum height attained by the bird?
- What is the range of the bird missile?

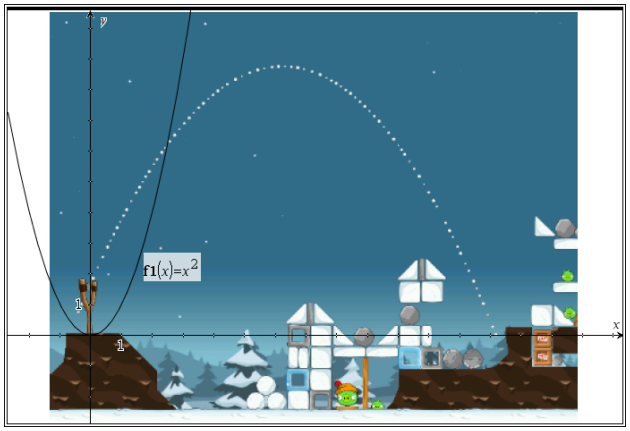
Part 1

In order to investigate this problem we need to set the origin of the coordinate axes to an appropriate reference point in the graphic, i.e. the base of the catapult. Move the cursor arrow over the origin, type ctrl and a hand will appear which allows you to move the axes to the base of the catapult.

Enter the function $f1(x) = x^2$. Move the cursor to the minimum point of the graph and the cursor

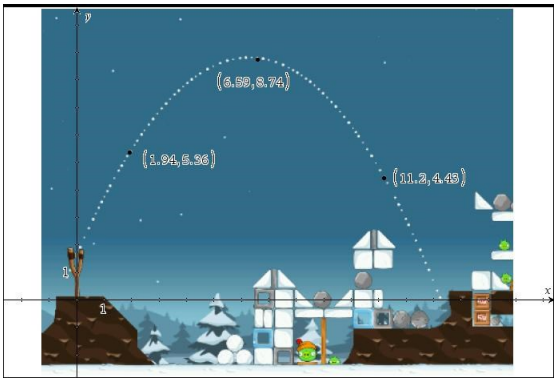
will become a cross. Grab the graph (ctrl) and translate it so that it at the maximum point of the trajectory of the angry bird. Move the cursor to the sloping sides of the parabola and the cursor will become a diagonal cross. Grab the graph again and this will stretch the graph so that it fits over the trajectory. The equation of the transformed parabola will be displayed on the screen and can be used to answer the questions posed at the start of the activity.



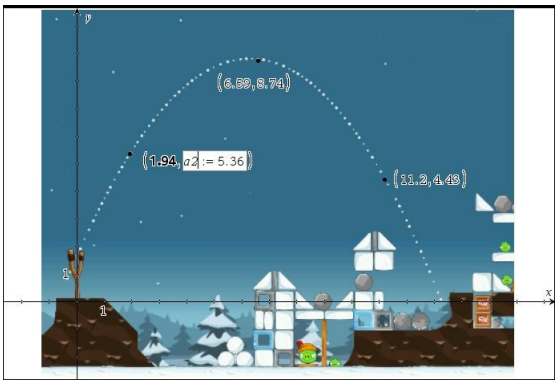


Part 2

Create a new graphics page with the angry bird graphic background. Move the origin of the axes as before. Place three well-spaced points on the trajectory curve: menu 8: Geometry – 1: Points and lines – 1: Points. Obtain the coordinates of the three points and write them on the graphics screen: menu 1: Actions – 8: Coordinates and equations.

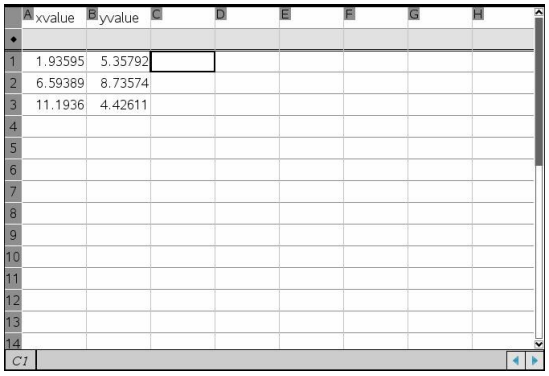


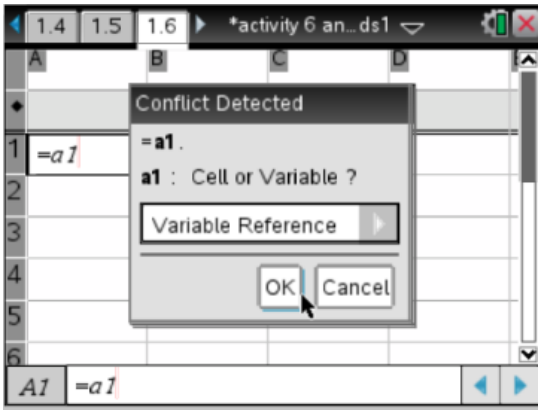
For each x and y coordinate of each of the three points move the cursor over the text, press "shift menu 5: Store" and define a variable for each number. Use (*a1*,*a2*), (*b1*,*b2*) and (*c1*,*c2*) as the variables for the three coordinates.



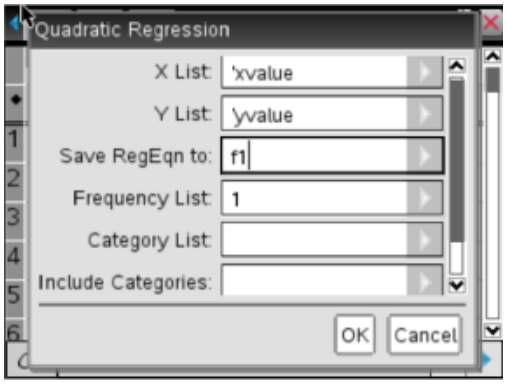
Create a spreadsheet page and enter the coordinates onto the spreadsheet. Put the x coordinates in column A and the y coordinates in column B. Name the two columns xvalue and yvalue .

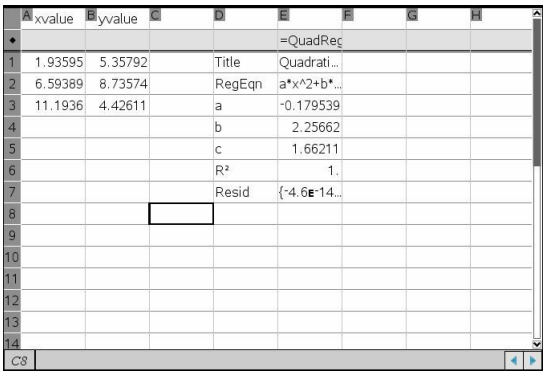
There will be a conflict detected about the variable names, select Variable Reference.





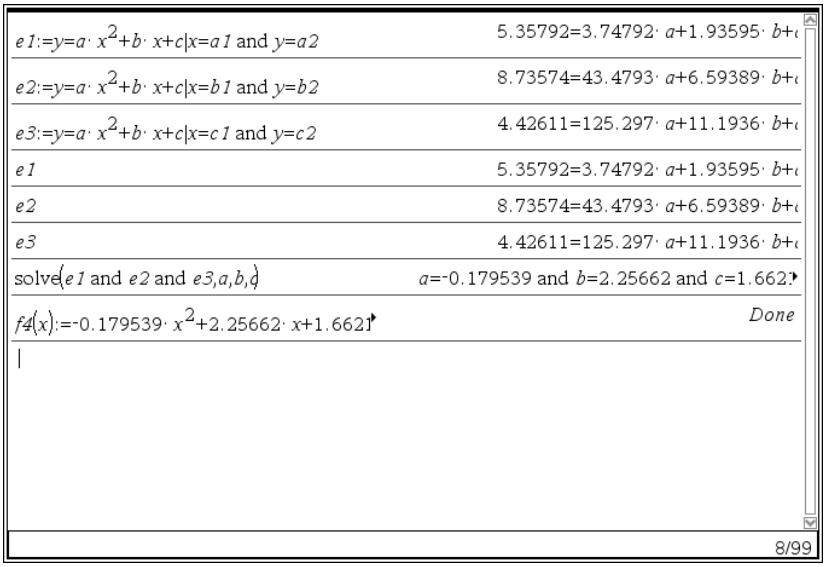
Use the coordinates in the spreadsheet to calculate a quadratic regression line using the command sequence: menu 4: Statistics – 1: Stats calculations – 5: Quadratic Regression as shown in the graphic below.





Part 3

An alternative strategy is to use the three coordinates points to set up a system of three simultaneous equations to obtain the quadratic equation for the trajectory in the form $y = ax^2 + bx + c$. To do this open a calculator page and follow the instructions in the graphic below.

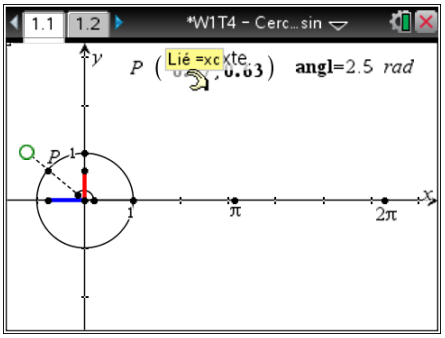


Activity 6: Scattergraphs, sine, cosine and tangent functions

Open the *tns* file "*W6 – Trigonometry*".

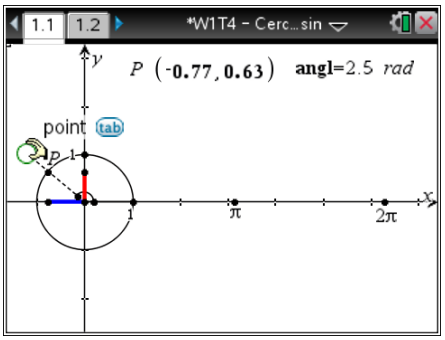
In the graphics window we represent a trigonometric circle, an angle measured in radians taking values from 0 to 2π and a point *P* on the circle with coordinates (*xc* ; *yc*).

The coordinates of *P* are stored as variables in the calculator: the *x* coordinate of *P* is stored in the variable *xc* and its *y* coordinate in *yc*.



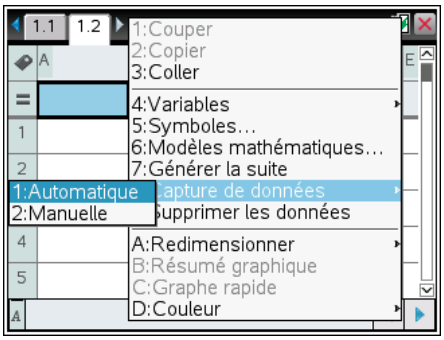
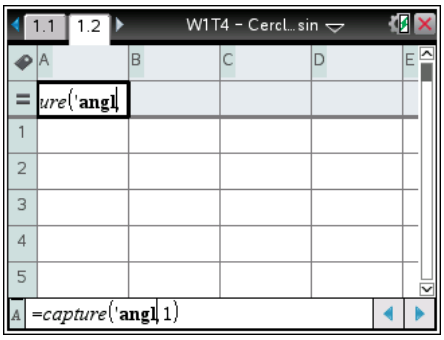
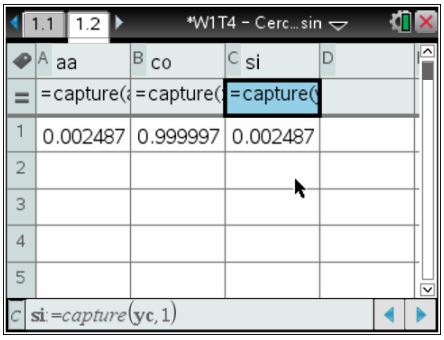
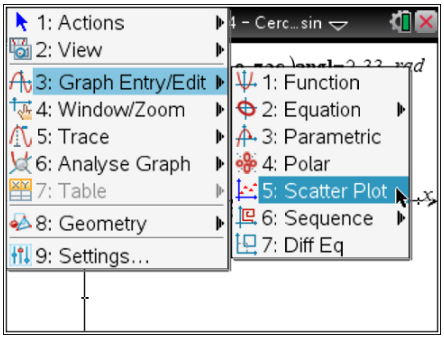
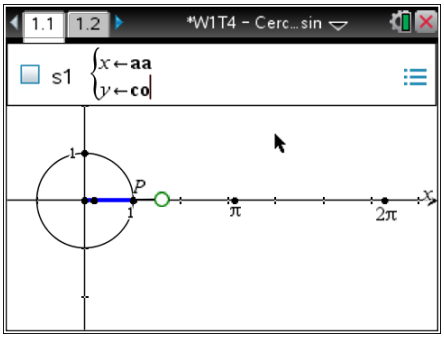
Observe the coordinates of *P* for different values of angle as you move the point *P*.

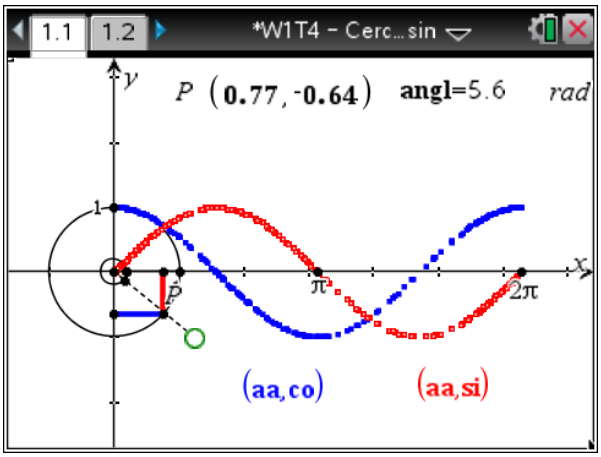
Then replace the point in the first quadrant.



In a spreadsheet we are going to capture the coordinates of *P* for different values of the angle.

We are also going to obtain two scattergraphs : Nuage : (*angl*,*xc*) and Nuage2 : (*angl*,*yc*).

Open a “Lists & Spreadsheet” page. Place the cursor in the second row of the first column, then: “ctrl menu 8: Data capture – 1: Automatic”.	
Choose **angl** (in var). Do the same in the 2nd and 3rd columns with **xc** and **yc** respectively.	
Give a name to each column (for example: aa, co et si). Careful: the names of pre-defined commands in the calculator are not allowed.	
Return to the graph page: “menu 3: Graph Entry/Edit – 5: Scatter plot”.	
Choose for *x* the variable **aa** and for y the variable **co** then enter. For the second scatter graph, ctrl G, choose for *x* the variable **aa** and for *y* the variable **si** then enter. Then make the angle change.	



Activity 7: Sliders and transformation of graphs

Open the tns file "*W7 - Transformation of graphs game*" and follow the instructions.