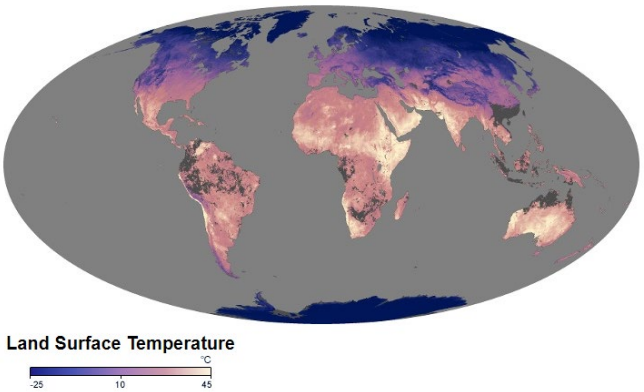
1. Calculating the effective temperature of the Earth T_E



To calculate the effective temperature of the Earth, the energy received from the Sun is equated to the energy radiated by the Earth, using a black-body approximation.

The power, E_S, emitted by the Sun emitted by the Sun using the Stefan-Boltzmann law is given by:

$$E_S = 4\pi r_S^2 \sigma T_S^4$$

At Earth, this energy is passing through a sphere with a radius of a_0, the distance between the Earth and the Sun, and the energy, E_{a0}, passing through each square metre of the sphere is given by

$$E_{a_0} = \frac{E_S}{4\pi a_0^2}$$

The Earth has a radius of r_E, and therefore has a cross-section of πr_E^2. The amount of solar power absorbed by the Earth is thus given by:

$$E_{abs} = \pi r_E^2 \times E_{a_0}$$

The amount of energy emitted must equal the amount of energy absorbed, and so, again using the Stefan-Boltzmann law:

$$\begin{aligned} 4\pi r_E^2 \sigma T_E^4 &= \pi r_E^2 \times E_{a_0} \\ &= \pi r_E^2 \times \frac{4\pi r_S^2 \sigma T_S^4}{4\pi a_0^2} \end{aligned}$$

T_E can then be found:

$$T_E^4 = \frac{r_S^2 T_S^4}{4a_0^2}$$

$$T_E = T_S \times \sqrt{\frac{r_S}{2a_0}}$$

$$= 5780\ \mathrm{K} \times \sqrt{\frac{696 \times 10^6\ \mathrm{m}}{2 \times 149.598 \times 10^9\ \mathrm{m}}}$$

$$\approx 279\ \mathrm{K}$$

T_E = 6 °C

where T_S is the temperature of the Sun, r_S the radius of the Sun, and a_0 is the distance between the Earth and the Sun. This gives an effective temperature of 6°C on the surface of the Earth, assuming that it perfectly absorbs all emission falling on it and has no atmosphere so perfectly emits.

However, the Earth has an albedo and cloud cover. Cloud cover evidently contributes to the albedo, but it also inhibits longer wavelength radiation being emitted from the Earth. In the following section, the model used for calculating the mean temperature of the Earth is modified by the albedo and cloud cover.

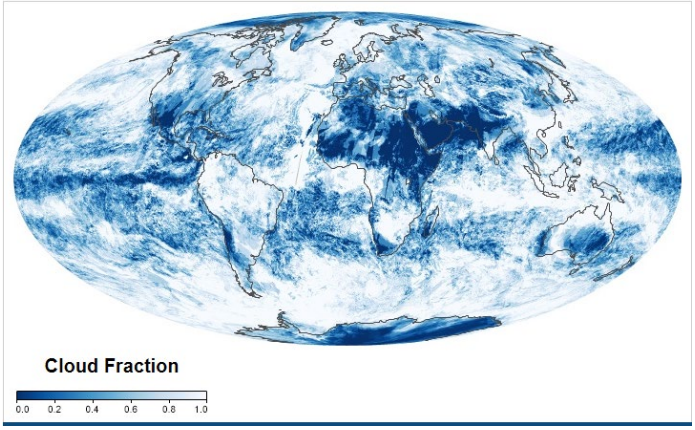
The albedo can be modelled in the school classroom using sheets of coloured paper and light sensors. https://resources.t3europe.eu/t3europe-home?resource_id=3103 .

It is approximately 0.3 meaning that the Earth emits 30% of the radiation falling upon it and therefore absorbs just 70%. On its own this would reduce the mean temperature of the Earth to -18 °C, distinctly cold. At this temperature the Earth's oceans would be frozen solid meaning there would be no life on this planet.

Fortunately, the clouds are made of water and water vapour which is a very good absorber of long wavelength infrared radiation. The assumption is made that the clouds absorb all the radiation coming from the Earth and re-radiate randomly in all directions, meaning that only half of the radiation impacting the clouds is emitted. Cloud cover has been measured for a long time and is considered to be about 68%.

The next section recalculates the Earth's mean temperature based on these modifications.

2. Albedo, the atmosphere and cloud cover

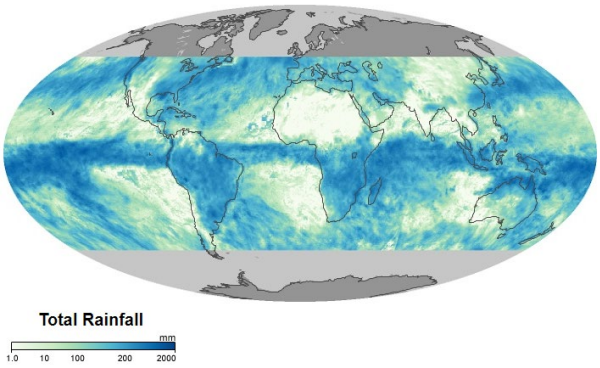


The Earth's albedo is approximately 0.29. This means that seen from outer space the Earth reflects 29% of the radiation falling on it from the Sun. Measuring the albedo is a long and painstaking affair using satellite data and of course human beings are steadily changing the albedo through construction work and forest clearance. An experiment to explore the Earth's albedo can be found at https://resources.t3europe.eu/t3europe-home?resource_id=3103 .

This means that only 71% of the incoming radiation is absorbed. All other things being equal this would mean that the mean global temperature would be 256 K which is well below freezing meaning there would be no life on this planet! One of the contributors to the albedo is cloud cover and this is discussed below.

Fortunately, the Earth has an atmosphere which contains water vapour and in particular clouds which are dense concentrations of the vapour in the form of water droplets and vapour. It is of course from these clouds that water falls as rain making its way back to the oceans from where it is evaporated to reform the clouds. Again, satellite data reveal that the Earth's cloud cover is fairly consistent at about 68%. Of course, this varies from time to time and place to place. The NASA Earth Observatory website has a good graphic showing this variation at https://earthobservatory.nasa.gov/global-maps click on cloud fraction.

Humans are increasing the cloud cover resulting in increased rainfall in certain areas and drought in others.



See https://resources.t3europe.eu/t3europe-home?resource_id=3111 .

The radiation emitted by the Earth is absorbed by water vapour in the atmosphere and we may make the assumption that radiation from the Earth is totally absorbed by clouds so that half is re-radiated back to Earth and the other half escapes through the upper atmosphere. This means that the radiation escaping is one half of 68% plus 32% or a total of 66%.

It is now possible to include the effects of albedo and cloud cover in the original model.

$$T_E = \left(\sqrt[4]{\frac{0.71}{0.66}} \right) 279 \text{ K}$$

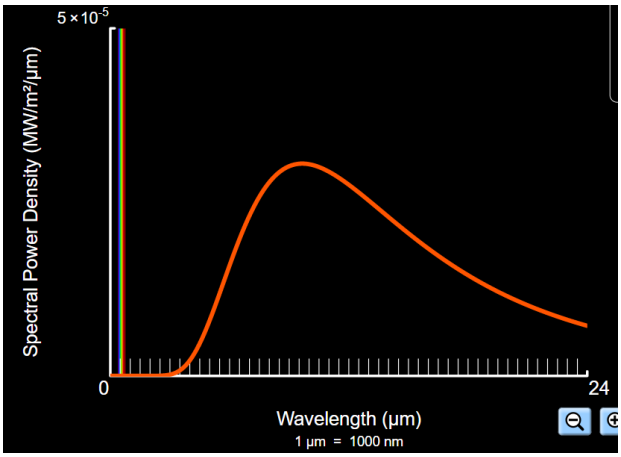
$$\mathbf{T_E = 284 \text{ K or } 11 \text{ °C}}$$

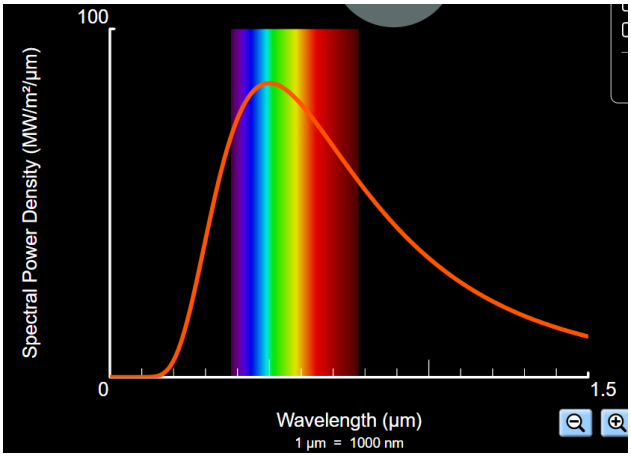
These two simple modifications to the original model provide a mean global temperature of 11 °C. The accepted value today is 14 °C which shows that the model is moving in the right direction.

These two modifications are concerned with an environment which is not affected by human interference. They represent what is called the Greenhouse effect but take no account of the effect of absorption of outgoing radiation by for example carbon dioxide in the atmosphere. The anomalous greenhouse effect is caused by the injection into the atmosphere of, methane, additional CO_2 from burning fossil fuels and other aerosols which absorb infrared radiation.

3. A note on the qualitative difference between radiation from the Sun and the Earth

Graphs of the Black Body radiation (*PhET, credit below*) from the Sun (right) and Earth (left) reveal that the peak wavelength is far into the infrared for the Earth and well into the visible for the Sun. There is a great deal of short wavelength ultraviolet and short wavelength infrared from the Sun and the peak is in the visible spectrum at less than 1 micron. However, radiation from the Earth clearly peaks in the far infrared at about 10 microns with effectively nothing in the visible spectrum. This is because of their different temperatures, the Sun being at nearly 6000 K while the Earth is a very cool 287 K. Incoming infrared from the Sun is very short wavelength while outgoing infrared from the Earth is very long wavelength. It is this long wavelength radiation which is easily absorbed by certain gases in the atmosphere. Gases such as ozone are fortunately good at absorbing very short wavelength radiation such as ultraviolet which is so damaging to life on this planet. The balance is a fragile one and human activity is upsetting this balance resulting in enhanced global warming from additional CO_2 and the hole in the ozone layer from chlorofluorocarbons (CFCs). See https://resources.t3europe.eu/t3europe-home?resource_id=3152 .





4. Credits

https://earthobservatory.nasa.gov/global-maps credit NASA

Simulation by PhET Interactive Simulations, University of Colorado Boulder, licensed under CC-BY-4.0 (https://phet.colorado.edu).