The Bungee Jumper

Consider a bungee jumper on a platform with the cord coiled up beside them. The jumper's total energy is zero. As the jumper dives downwards, energy is shifted out of the gravitational store and into the elastic and kinetic stores. All the time the total energy E_{tot} must be zero. We will assume that none of the energy is shifted into the thermal store and find out what the total of the three energy stores is for values of x. We need to write all three energy stores as a function of x.

The jumper mass m reaches the point where the cord length L has just become taut. Here x=0. The cord begins to stretch so that,

$$E_e = ½kx^2 \text{ and}$$

$$E_g = -mgL-mgx$$

The cord will stretch and reach the equilibrium position x_e where the tension in the cord, elastic constant k, is equal to the weight of the jumper

$$mg = kx_e \quad \text{..........................(1)}$$

We need the kinetic energy as a function of x. But from the position x=0 the acceleration is no longer uniform so the equations of uniform motion cannot be used. The equations of simple harmonic motion can be used here, so that v the velocity of the jumper is given by

$$v = \sqrt{(k/m)}\sqrt{(A^2 – (x-x_e)^2)}$$

where $A + x_e$ is the maximum stretch of the cord when the jumper is at rest at the lowest point. That is at the lowest point

$$x = A + x_e$$

We can now write the kinetic energy E_k as a function of x

$$E_k = ½ m(k/m)(A^2 – (x-x_e)^2)$$

When x=0, $E_k = E_g$ lost

$$\therefore mgL = k/2 (A^2 – x_e^2) \quad \text{....................(2)}$$

$$E_{tot} = E_g + E_e + E_k$$

$$E_{tot} = -mgL -mgx + ½kx^2 + ½k(A^2 – x^2 + 2xx_e – x_e^2)$$

$$= -mgL –kx_e x + ½kx^2 + ½kA^2 – ½kx^2 + kxx_e – ½kx_e^2 \text{ .. from (1)}$$

$$= -mgL + ½kA^2 - ½kx_e^2$$

$$= -mgL + k/2(A^2 – x_e^2)$$

$$= -mgL + mgL \quad \text{..from (2)}$$

$$= 0$$

So E_{tot} is zero for all values of x, just as it should be!

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